# On correspondence between right near-domains and sharply 2 -transitive groups 

Andrey A. Simonov<br>a.simonov@g.nsu.ru


#### Abstract

The right near-domain is defined to loosen neardomain axioms. Correspondence of a class of the right near-domains and a class of sharply 2 -transitive groups is constructed.


Keywords: near-domain, sharply 2-transitive groups.
In [1, 2] for exposition of sharply 2-transitive groups the concept near-domain is introduced. Near-domain is an algebraic system $\left(B, \cdot,+,^{-1}, 0\right)$ with two binary operations $\cdot,+$ for which axioms hold:

1. $(B,+)$ is a loop with a unit element 0 ;
2. $a+b=0 \Rightarrow b+a=0$;
3. $\left(B_{1}, \cdot,{ }^{-1}\right)$ is a group with an unit element $e$, where $B_{1}=B \backslash\{0\}$;
4. $(\forall x \in B) \quad x \cdot 0=0$;
5. $(\forall x, y, z \in B) \quad(x+y) \cdot z=x \cdot z+y \cdot z$;
6. $(\forall a, b \in B)\left(\exists r_{a, b} \in B_{1}\right) \quad(x+a)+b=x \cdot r_{a, b}+(a+b)$ for any $x \in B$.

Until recently it is not known any example of a near-domain which is not a near-field. In the given work it is offered to loosen near-domain axioms, having left only necessary ones for construction of sharply 2 -transitive groups. In particular, it is offered to refuse from axioms 2,4 and to loosen axioms 1,5 .

Let's define the right near-domain as algebraic system $\left(B, \cdot,+,-,{ }^{-1}, 0\right)$ with operations: $(+): B \times B_{1} \rightarrow B,(-): B \times B_{1} \rightarrow B,(\cdot): B \times B_{1} \rightarrow B$, where $B_{1}=B \backslash\{0\}$, for which axioms are hold
A1. $(\forall x \in B)\left(\forall y \in B_{1}\right)(x-y)+y=x$;
A2. $(\forall x \in B)\left(\forall y \in B_{1}\right)(x+y)-y=x$;
A3. $\left(\forall x \in B_{1}\right) \quad x-x=0$;
A4. $\left(B_{1}, \cdot,{ }^{-1}\right)$ is a group with an unit element $e \in B_{1}$;
A5. $(\forall x \in B)\left(\forall y, z \in B_{1}\right)\left(\exists h(y, z) \in B_{1}\right)(x+y) z=x h(y, z)+y z$;
A6. $(\forall x \in B)\left(\forall y, z \in B_{1}: y+z \neq 0\right)\left(\exists r(y, z) \in B_{1}\right)(x+y)+z=x r(y, z)+$ $(y+z)$;
A7. $(\forall x \in B)\left(\forall z \in B_{1}\right)\left(\exists v(z) \in B_{1}\right)(x+(0-z))+z=x v(z)$.
Axioms A1-A3 define algebraic system $(B,+,-, 0)$ as the right loop. We define labels $L(x)=0-x$ then from A1 follows $L(x)+x=0$. Thus the map $L: B_{1} \rightarrow B_{1}$ defines left inverse in the right loop.

Let's consider now the elementary consequences of axioms.
Lemma. In the right near-domain the following properties hold:

1. $\left(\forall x \in B_{1}\right) 0 x=0$;
2. $h(x, y)=E L(x) L(x y)$, where $E(x)=x^{-1}, E L$ - superposition of transformations $L$ and $E$;
3. $r(y, z)=E(L(z)-y) L(y+z)$;
4. $x-z=x E v(z)+L(z)$;
5. $v(z)=E L^{2}(z) z$

Let's define a map $u: B_{1} \rightarrow B$ by the rule $u(x)=0 x$.
From A5 follows, that $\left(\forall x, y \in B_{1}\right)(L(x)+x) y=L(x) h(x, y)+x y=u(y)$, hence

$$
\begin{equation*}
h(x, y)=E L(x)(u(y)-x y) . \tag{1}
\end{equation*}
$$

If we will sequentially apply A 5 for arbitrary $z, t \in B_{1}$ then we receive:

$$
h(y, z) h(y z, t)=h(y, z t)
$$

Let's write the given equality applying the expression (1). With the reduction account, we will receive equality : $(u(z)-y z) E L(y z)=e$, hence, $u(z)=L(y z)+$ $y z=0$. Thus, the first and second conditions of the lemma are satisfied.

Let's consider now consequences from A6. Let $x=L(y+z)(r(y, z))^{-1} \Rightarrow$ $\left(L(y+z)(r(y, z))^{-1}+y\right)+z=0$, whence we will receive the expression from the third condition of the lemma.

In the case when $y+z=0$, we will consider consequences from A7 and A2: $x+L(z)=x v(z)-z$. We define $x^{\prime}=x v(z)$, hence the fourth condition of the lemma is fulfilled $x^{\prime} E(v(z))+L(z)=x^{\prime}-z$.

Let's note A2 with the condition of the received expression $(x+z)-z=$ $(x+z) E v(z)+L(z)=x$. At $x=0$ we will receive equality $z E v(z)+L(z)=0$. Then with the account $L^{2}(x)=L L(x)$, we will come to justice of the fifth condition of the lemma.

The operation " -" is expressed through the operations " + ", ".", $L, E$ hence we will understand algebraic system $\left(B, \cdot,+,-,^{-1}, 0\right)$ as $\left(B, \cdot,+,{ }^{-1}, L, 0\right)$.

Let's consider the algebraic system $\left(H, \cdot, \phi,{ }^{-1}, 0\right)$ from [3] with the operations:

$$
(\cdot): H \times H_{1} \rightarrow H, \phi: H \rightarrow H, \text { where } H_{1}=H \backslash\{0\}
$$

for which the following axioms are fulfilled:
F1. $\left(H_{1}, \cdot,^{-1}\right)$ is a group with an unit element $e$;
F2. $0 x=0, x \in H_{1}$;
F3. $\phi(e)=0$;
F4. $\phi(\phi(x) \phi(y))=\phi\left(x \phi\left(y^{-1}\right)\right) y, x \in H, y \in H_{1} \backslash\left\{e_{1}\right\}$,
The similar algebraic system was investigated in 4].
Theorem 1. The class of algebraic systems $\left(B, \cdot,+,{ }^{-1}, L, 0\right)$ and $\left(B, \cdot,^{-1}, \phi, 0\right)$ are rational equivalent.
Let's introduce a map $\phi: B \rightarrow B$, defined in an aspect $\phi(x)=x(0-e)+e=$ $x a+e$. Let's calculate quadrate of function $\phi$ taking into account the conditions two and five of the lemma:

$$
\phi^{2}(x)=(x a+e) a+e=(x L(a)+a)+e=x L(a) E L^{2}(e)=x
$$

From the definition follows $\phi(e)=a+e=0$ and $\phi(0)=e$. By means of the $\operatorname{map} \phi$ it is possible to express additive operation. Really, $\phi(x) y=(x a+e) y=$ $x L(y)+y$, hence, if $x=z E L(y)$, then $z+y=\varphi(z E L(y)) y$. Let's rewrite now
identity from A2: $z=(z+y)-y=\phi(z E L(y)) y-y$. Having introduced labels $t=\phi(z E L(y)) y$, we express $z=\phi\left(t y^{-1}\right) L(y)$, then $t-y=\phi\left(t y^{-1}\right) L(y)$.

Calculating the value $t=(x+z)-(y+z)$ in the case $y \neq L(z)$, using at first A2: $(x+z)=t+(y+z)$, and then the third identity of the lemma: $(x+z)=\left(t(r(y, z))^{-1}+y\right)+z$. Applying twice identity from A2, we have the identity:

$$
(x+z)-(y+z)=(x-y)(L(z)-y)^{-1} L(y+z)
$$

Let's rewrite the given identity with the account $y \neq e, z=L^{-1}(e)$ replacing additive binary operations by their expressions through the function $\phi$ :

$$
\begin{equation*}
\phi(\phi(x) E \phi(y))=\phi\left(x y^{-1}\right) E \phi E(y)=\phi\left(x y^{-1}\right) E \phi E(y) \tag{2}
\end{equation*}
$$

At $x=0$ this identity takes a simple form $\phi E \phi(y)=E \phi E(y)$, using it, we note identity (2) for $y=E \phi E(t)$ :

$$
\begin{equation*}
\phi(\phi(x) \phi(t))=\phi(x \phi E(t)) t \tag{3}
\end{equation*}
$$

Thus, we have the map $\mathbb{A}:\left(B, \cdot,+,^{-1}, L, 0\right) \rightarrow\left(B, \cdot,^{-1}, \phi, 0\right)$.
Let's make the inverse construction. We will consider expression from F4 at $x=e, y=t^{-1}$, then under condition F2 and F3 we come to equality $\varphi^{2}(t)=\varphi(0) t$. On one hand $\varphi^{4}(t)=(\varphi(0))^{2} t$, and on the other hand $\varphi^{4}(t)=$ $\varphi\left(\varphi^{2}(\varphi(t))\right)=\varphi(\varphi(0) \varphi(t))$. It is also possible to note the last expression with the account F4 and F2: $\varphi(\varphi(0) \varphi(t))=\varphi\left(0 \varphi\left(t^{-1}\right)\right) t=\varphi(0) t$. Thus, we come to equality $\varphi^{2}(0)=\varphi(0)$, hence, $\varphi(0)=e$ and $\varphi^{2}(t)=t$.

From F4 for $x=E \varphi E(y)$ follows, that $\left(\forall y \in B_{1} \backslash\{e\}\right) \varphi E \varphi(y)=E \varphi E(y)$.
By means of arbitrary bijection $L: B_{1} \rightarrow B_{1}$ we introduce operations

$$
x+y=\varphi(x E L(y)) y, x-y=\varphi\left(x y^{-1}\right) L(y)
$$

With the account of F2, F3 and $\varphi^{2}=i d$ it is easy to check up the performance of the axioms A1-A3 of the right loop. The performance A5 follows from the operation definition:

$$
(x+y) z=\varphi(x E L(y)) y z=\varphi(x E L(y) L(y z) E L(y z)) y z=x E L(y) L(y z)+y z
$$

Then we take advantage of identities $\varphi^{2}=i d, \varphi E \varphi=E \varphi E$ and F 4 to receive A6:

$$
\begin{gathered}
(x+y)+z=\varphi(\varphi(x E L(y)) y E L(z)) z= \\
\varphi(x E L(y) \varphi E \varphi(y E L(z))) \varphi(y E L(z)) z= \\
\varphi(x E L(y) \varphi E \varphi(y E L(z)) L[\varphi(y E L(z)) z] E L[\varphi(y E L(z)) z]) \varphi(y E L(z)) z= \\
x E L(y) \varphi E \varphi(y E L(z)) L[\varphi(y E L(z)) z]+(y+z)= \\
x E(\varphi(L(z) E(y)) L(y)) L[\varphi(y E L(z)) z]+(y+z)=x E(L(z)-y) L(y+z)+(y+z) .
\end{gathered}
$$

Now we take advantage of identity $\varphi^{2}=i d$ for construction of expression A7:

$$
(x+L(z))+z=\varphi\left(\varphi\left(x E L^{2}(z)\right) L(z) E L(z)\right) z=\varphi^{2}\left(x E L^{2}(z)\right) z=x E L^{2}(z) z
$$

For any bijection $L$ we have constructed the $\operatorname{map} \mathbb{F}_{L}:\left(B, \cdot,^{-1}, \varphi, 0\right) \rightarrow\left(B, \cdot,+,^{-1}\right.$, $L, 0)$ so, that the algebraic systems $\left(B, \cdot,^{-1}, \varphi, 0\right)$ and $\mathbb{A} \circ \mathbb{F}_{L}\left(B, \cdot,^{-1}, \varphi, 0\right)$ are isomorphic. In the opposite direction the algebraic systems $\left(B, \cdot,+^{\prime},{ }^{-1}, L^{\prime}, 0\right)$ and $\mathbb{F}_{L} \circ \mathbb{A}\left(B_{1}, 0, L^{\prime}, \cdot,+^{\prime}\right)$ are isomorphic only at $L=L^{\prime}$.

The group $T_{2}(B)$ of transformations of a set $B$ is called sharply 2 -transitive group, if for arbitrary pairs $\left(x_{1}, x_{2}\right) \neq\left(y_{1}, y_{2}\right) \in \widehat{B^{2}}$, where $\widehat{B^{2}}=B^{2} \backslash\{(x, x) \mid x \in$ $B\}$ there exists an unique element $g \in T_{2}(B)$ for which the equalities $g\left(x_{1}\right)=y_{1}$ and $g\left(x_{2}\right)=y_{2}$ are held.

Theorem 2. The class of algebraic systems $\left(B, \cdot,^{-1}, \varphi, 0\right)$ and the class of sharply 2-transitive groups $T_{2}(B)$ are rational equivalent.
On the set $\widehat{B^{2}}$ we define a function $f: B \times \widehat{B^{2}} \rightarrow B$ as

$$
\begin{equation*}
f\left(x, y_{1}, y_{2}\right)=\varphi\left(x \varphi\left(y_{1} y_{2}^{-1}\right)\right) y_{2} \tag{4}
\end{equation*}
$$

if $y_{2} \neq 0$ and $f\left(x, y_{1}, 0\right)=x y_{1}$ otherwise. Not to consider two cases separately, we, by means of multiplicative partial operation $(\cdot): B \times B_{1} \rightarrow B$, define the groupoid on $B$ so, that $(\forall x \in B) x 0=\varphi(x), 0^{-1}=0$.

Let's define a binary operation $G$ on the set $\widehat{B^{2}}$ in the form

$$
\begin{equation*}
\binom{x_{1}}{x_{2}}\binom{y_{1}}{y_{2}}=\binom{f\left(x_{1}, y_{1}, y_{2}\right)}{f\left(x_{2}, y_{1}, y_{2}\right)}=\binom{\varphi\left(x_{1} \varphi\left(y_{1} y_{2}^{-1}\right)\right) y_{2}}{\varphi\left(x_{2} \varphi\left(y_{1} y_{2}^{-1}\right)\right) y_{2}} . \tag{5}
\end{equation*}
$$

Supposing, that there are pairs $\left(x_{1}, x_{2}\right) \neq\left(y_{1}, y_{2}\right) \in \widehat{B^{2}}$, that $f\left(x_{1}, y_{1}, y_{2}\right)=$ $f\left(x_{2}, y_{1}, y_{2}\right)$. Then, for $y_{2} \neq 0$ after multiplication of the both parts of equality on $y_{2}^{-1}$ and transformations by the function $\varphi$, we will come to equality $x_{1} \varphi\left(y_{1} y_{2}^{-1}\right)=x_{2} \varphi\left(y_{1} y_{2}^{-1}\right)$ from which follows, that $x_{1}=x_{2}$. At $y_{2}=0$ we get the equality $x_{1} y_{1}=x_{2} y_{1}$, hence, $x_{1}=x_{2}$. We have come to an inconsistency. Thus, the operation $G$, defined above, is a magma.

It is easy to check, that the pair $(e, 0) \in \widehat{B^{2}}$ is the left and the right unit element. Now we check the associativity:

$$
\begin{gathered}
\varphi\left(\varphi\left(x_{i} \varphi\left(y_{1} y_{2}^{-1}\right)\right) y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right) z_{2}=\varphi\left(\varphi\left(x_{i} \varphi\left(y_{1} y_{2}^{-1}\right)\right) \varphi \varphi\left(y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right)\right) z_{2}= \\
\varphi\left(x_{i} \varphi\left(y_{1} y_{2}^{-1}\right) \varphi E \varphi\left(y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right)\right) \varphi\left(y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right) z_{2}= \\
\varphi\left(x_{i} \varphi\left(y_{1} \varphi\left(z_{1} z_{2}^{-1}\right) E \varphi\left(y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right)\right) \varphi\left(y_{2} \varphi\left(z_{1} z_{2}^{-1}\right)\right) z_{2}\right.
\end{gathered}
$$

We have come to a semigroup with a unit element. We will discover now the left inverse:

$$
\binom{x_{1}}{x_{2}}^{-1}\binom{x_{1}}{x_{2}}=\binom{\varphi\left(x_{2}^{-1}\right) E \varphi\left(x_{1} x_{2}^{-1}\right)}{E \varphi\left(x_{1} x_{2}^{-1}\right)}\binom{x_{1}}{x_{2}}=\binom{e}{0}
$$

Let's check that it is also the right inverse:

$$
\varphi\left(x_{i} \varphi\left(\varphi\left(x_{2}^{-1}\right) E \varphi\left(x_{1} x_{2}^{-1}\right) \varphi\left(x_{1} x_{2}^{-1}\right)\right)\right) E \varphi\left(x_{1} x_{2}^{-1}\right)=\varphi\left(x_{i} x_{2}^{-1}\right) E \varphi\left(x_{1} x_{2}^{-1}\right)
$$

Thus, we defined that $G$ is a group, but since it operates on the set $\widehat{B^{2}}$ sharply transitive, then the group $G$ at an operation on the set $B$ it will be sharply 2 -transitive, hence, we have constructed the map $\mathbb{G}:\left(B, \cdot,^{-1}, \varphi, 0\right) \rightarrow T_{2}(B)$.

Now we make the inverse construction and on a group $T_{2}(B)$ we will construct an algebraic system $\left(B, \cdot,^{-1}, \phi, 0\right)$. For an arbitrary pair $\left(e_{1}, e_{2}\right) \in \widehat{B^{2}}$ it is possible to construct the bijective map $T_{2}(B) \rightarrow \widehat{B^{2}}$, putting in correspondence to an element $g \in T_{2}(B)$ the pair $\left[x_{1}, x_{2}\right]$ so that

$$
\begin{equation*}
\left(e_{1}, e_{2}\right) \cdot g=\left(e_{1}, e_{2}\right) \cdot\left[x_{1}, x_{2}\right]=\left(e_{1} \cdot\left[x_{1}, x_{2}\right], e_{2} \cdot\left[x_{1}, x_{2}\right]\right)=\left(x_{1}, x_{2}\right) \tag{6}
\end{equation*}
$$

The given bijection induces the isomorphic group $G \simeq T_{2}(B)$ on the set of pairs $\widehat{B^{2}}$. The pair $\left[e_{1}, e_{2}\right]$ is an unit of the group $G$.

At serial transformation of the pair $\left(e_{1}, e_{2}\right)$ by elements $\left[x_{1}, x_{2}\right]$ and $\left[y_{1}, y_{2}\right]$ we come to equality:

$$
\begin{equation*}
\left[x_{1}, x_{2}\right]\left[y_{1}, y_{2}\right]=\left[x_{1} \cdot\left[y_{1}, y_{2}\right], x_{2} \cdot\left[y_{1}, y_{2}\right]\right] \tag{7}
\end{equation*}
$$

from which, with the account (6), follows, that on a subset $B_{1}=\left\{x \in B \mid\left[x, e_{2}\right] \in\right.$ $G\}$ it is possible to introduce the group structure naturally. The map $e_{1} \cdot\left[x, e_{2}\right] \mapsto$ $x$ induces on $B_{1}$ a group structure. Multiplication in the group $B_{1}$, as well as in the group $T_{2}(B)$ we will write without a point. We will expand the group operation to a partial operation $B \times B_{1} \rightarrow B$, having predetermined it in an aspect $e_{2} y=e_{2} \cdot\left[y, e_{2}\right]=e_{2}$ so, that $e_{2}$ will be the left zero in a partial operation $(\cdot): B \times B_{1} \rightarrow B$.

From (6) and (7) follows, $\left[e_{2}, e_{1}\right]$ is an involution of $G$. We define $\phi: B \rightarrow B$ in an aspect $\phi(x)=x \cdot\left[e_{2}, e_{1}\right]$, then $\phi\left(e_{1}\right)=e_{2}$ and

$$
\begin{equation*}
\left[e_{2}, e_{1}\right]\left[x_{2}, x_{1}\right]=\left[x_{1}, x_{2}\right]=\left[\phi\left(x_{1}\right), \phi\left(x_{2}\right)\right]\left[e_{2}, e_{1}\right] \tag{8}
\end{equation*}
$$

For an arbitrary $\left[e_{1}, x_{2}\right] \in G$, at $x_{2} \in B_{1} \backslash\left\{e_{1}\right\}$ it is possible to note:

$$
\left[e_{1}, x_{2}\right]=\left[x_{2}^{-1}, e_{1}\right]\left[x_{2}, e_{2}\right]=\left[\phi\left(x_{2}^{-1}\right), e_{2}\right]\left[e_{2}, e_{1}\right]\left[x_{2}, e_{2}\right]
$$

On the other hand, with the account (8) for $\left[e_{1}, x_{2}\right]$ it is fair

$$
\left[e_{1}, x_{2}\right]=\left[e_{2}, e_{1}\right]\left[\varphi\left(x_{2}\right), e_{2}\right]\left[e_{2}, e_{1}\right] .
$$

Having taken advantage of the two received expressions and equating outcomes of transformation arbitrary $t \in B$ by element $\left[e_{1}, x_{2}\right] \in G$, we come to identity:

$$
\phi\left(\phi(t) \phi\left(x_{2}\right)\right)=\phi\left(t \phi\left(x_{2}^{-1}\right)\right) x_{2}, t \in B, x_{2} \in B_{1} \backslash\left\{e_{1}\right\}
$$

The map $\mathbb{F}_{\left(e_{1}, e_{2}\right)}: T_{2}(B) \rightarrow\left(B, \cdot,^{-1}, \phi, 0\right)$ is constructed, putting in correspondence to group $T_{2}(B)$ algebraic system $\left(B, \cdot,^{-1}, \phi, 0\right)$.

Let's notice still, that for arbitrary $\left[x_{1}, x_{2}\right] \in T_{2}(B)$ it is possible to note:

$$
\left[x_{1}, x_{2}\right]= \begin{cases}{\left[\phi\left(x_{1} x_{2}^{-1}\right), e_{2}\right]\left[e_{2}, e_{1}\right]\left[x_{2}, e_{2}\right],} & x_{2} \in B_{1} \\ {\left[x_{1}, e_{2}\right],} & x_{2}=e_{2}\end{cases}
$$

Then for arbitrary $t \in B$ under condition of $x_{2} \neq e_{2}$ and $t \cdot\left[x_{1}, e_{2}\right]=t x_{1}$ the equality:

$$
\begin{equation*}
t \cdot\left[x_{1}, x_{2}\right]=t \cdot\left[\phi\left(x_{1} x_{2}^{-1}\right), e_{2}\right]\left[e_{2}, e_{1}\right]\left[x_{2}, e_{2}\right]=\phi\left(t \phi\left(x_{1} x_{2}^{-1}\right)\right) x_{2} \tag{9}
\end{equation*}
$$

is fair. Comparing (4), (5) with (9) and (7) we come to that there is a natural isomorphism $\mathbb{G} \circ \mathbb{F}_{\left(e_{1}, e_{2}\right)}: T_{2}(B) \rightarrow T_{2}^{\prime}(B)$, thus $\mathbb{G} \circ \mathbb{F}_{\left(e_{1}, e_{2}\right)}=i d$. Isomorphism of algebraic systems $\mathbb{F}_{\left(e_{1}, e_{2}\right)} \circ \mathbb{G}:\left(B, \cdot,{ }^{-1}, \phi, 0\right) \rightarrow\left(B^{\prime}, .^{\prime},{ }^{-1}, \phi, e_{2}\right)$ is set by map $\mathbb{F}_{\left(e_{1}, e_{2}\right)} \circ \mathbb{G}: x \mapsto \varphi\left(x \varphi\left(e_{1} e_{2}^{-1}\right)\right) e_{2}$, thus $\mathbb{F}_{\left(e_{1}, e_{2}\right)} \circ \mathbb{G}=i d$.

Let's consider some examples of the right near-domains constructed over a skew field $\mathbb{K}$ for which $\varphi(x)=-x+1, x \in \mathbb{K}$. As the first example we consider $L(x)=a x$ :

$$
x \oplus y=-x a^{-1}+y, x \ominus y=-x a+a y, r(y, z)=-a^{-1}, v(z)=a^{-2}
$$

In such right near-domain bilaterial distributivity is fulfilled and the identity $L(x \oplus y)=L(x) \oplus L(y)$ is hold. For the second example over a skew field we consider $L(x)=-x^{-1}$, then
$x \oplus y=x y^{2}+y, x \ominus y=x y^{-2}-y^{-1}, r(y, z)=y^{2} z(z+y)^{-1}(y z+1), h(y, z)=z^{-1}$.
For the given right loop $L(x \oplus y) \neq L(x) \oplus L(y)$, but it is fulfilled $L(x) \oplus x=$ $x \oplus L(x)=0$.

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