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Mathematical basis of physical structures theory

Abstract. Physics and mathematics progress find minimum means for the characterization of as much as possible wide class of phenomena. The original attempt of the reconsideration and the unification of physical laws were proposed by Kulakov. This area was developed by physicists. However it didn't attention among mathematicians. We will describe mathematical basic of the physical structures and some its results. First, we will consider the easiest examples of physical structures (Newton law, Ohm law) with using of Kulakov's method. Then we will determine the physical structure notion as the formalization of the examined examples. Subsequently general properties of physical structures will be obtained. Besides we consider generalizations of this theory. This is the first part of our analysis.

Keywords: Structures, Equations, Physical laws, Geometric laws.

Introduction

Physics and mathematics progress find minimum means for the characterization of as much as possible wide class of phenomena. The original attempt of the reconsideration and the unification of physical laws was proposed by Kulakov. It based on the physical structures theory (see [1 – 10]). The physical structures theory sets the large complex of the profound and nonstandard mathematical problems. The serious results in this domain were obtained by Kulakov (see [3, 5, 7, 10]), his pupils Mikhaylitchenko (see [11 – 26]), Lev (see [27, 28]), Simonov (see [29, 30]), and other researchers (see [31 – 41]). This area was developed by physicists. However it didn't attention among mathematicians. We will describe mathematical basic of the physical structures and some its results. First we will consider the easiest examples of physical structures (Newton law, Ohm law) with using of Kulakov's method. Then we will determine the physical structure notion as the formalization of the examined examples. Subsequently general properties of physical structures will be obtained. The final part of this paper is generalizations of this theory.

1. Physical structure of mechanical movements

Consider a rectilinear mechanical movement. A body moves under the influence of an external effect with changing its position and velocity. This phenomenon is described by Newton law $F = ma$, where F is a force, m is a mass, and a is an acceleration. It is possible to obtain the value of acceleration (general characteristic of this process) after measuring space and time intervals. However a method of measuring of the force and the mass is not obvious.

Question 1.1. *Is it possible to verify the applicability of Newton law with using measurable space and time characteristics only*?

It is necessary to exclude the force and the mass from Newton law for verification of its applicability. However two unknown values m and F with unique miserable parameter a affiliate to this equality. Then it is not sufficient to use two physical objects (movable object and accelerator) for resolution of this problem. If we have two different bodies and one accelerator, we receive

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two equations $F/m_1 = a_1$ and $F/m_2 = a_2$ with measurable accelerations a_1, a_2 and three unknown values m_1, m_2, and F. But if we have two different bodies and two different accelerators, we get four equations

$$F_j / m_k = a_{jk}, \quad j,k = 1,2 \qquad (1.1)$$

with four unknown values m_1, m_2, F_1, F_2, and four measurable accelerations a_{ij}.

We obtain the equality

$$a_{11}a_{22} - a_{12}a_{21} = 0 \qquad (1.2)$$

by formulas

$F_1 = m_1 a_{11} = m_2 a_{12}$, $F_2 = m_1 a_{21} = m_2 a_{22}$.

The measurable values only satisfy this equality.

Conclusion 1.1. *Two bodies and two accelerators are sufficient for the experimental verification of Newton law; then four measurable accelerations can be used for the verification of the equality* (1.2).

Formula (1.2) can be interpreted as a special form of Newton law. It can be experimentally verify directly. It is important that this equality is true for arbitrary pairs of physical objects. It characterizes a universal physical law.

Conclusion 1.2. *Four accelerations, characterized the relations arbitrary pairs of bodies and accelerators, are not unconditioned; one of them depends from others by equality* (1.2).

Conclusion 1.3. *It is not necessary to measure any masses and forces for verify Newton law; we do not use these notions even.*

Now we would like to use this result for analysis of the characteristics of our objects, i.e. the mass and the force.

Question 1.2. *How we can find the mass and the force with using measurable accelerators only?*

The system of linear homogeneous equations $F_j - a_{jk}m_k = 0, j,k = 1,2$ with four unknown values (two masses and two forces) has the zero determinant by (1.2). Then this problem has an infinity set of solutions. We get $m_2/m_1 = a_{11}/a_{12}$ because of $m_1 a_{11} = F_1 = m_2 a_{12}$. Choose the value m_1 as the unit (the mass standard) and determine the values a_{11} and a_{12} by experiment. Then we can find the comparative mass m_2, i.e. the value of this mass in comparison with the given standard. The force can be found analogically. We obtain $F_2/F_1 = a_{21}/a_{11}$ by $F_1/a_{11} = m_1 = F_2/a_{21}$. The value F_1 can be chosen as the force standard. Then we can find the comparative value on the force in comparison with that standard.

Conclusion 1.4. *The force and the mass can be determined by means of the measurable accelerations to within a choice of units.*

Remark 1.1. We had four equations (1.1) with four unknown values. Three equations of them only are independent because of the equality (1.2). Then the system (1.1) includes three conditions for four unknown values in really. The values F_1 and m_1 are chosen as the standard. The relevant equality $m_1 a_{11} = F_1$ does not connect any unknown values. So it can be excluded from the system. Then we have three equalities (1.1) with two unknown values m_2 and F_2. One of the equality is not independent by (1.2). Therefore there unknown values can be definite by standards and measurable values.

We supposed that the standard form of Newton law is known.

Question 1.3. *Is it possible to restore Newton law with using the relation between bodies and accelerators only?*

This is an inverse problem. It is necessary to determine the relation between a body i and an accelerator α with using the following assumption. It is sufficient to have two bodies and two accelerators and make four experiments for obtaining the required law. Then we get the values

$$\varphi(\alpha_j, i_k) = a_{jk}, j,k = 1,2. \qquad (1.3)$$

So the body i_k receives the acceleration a_{jk} by accelerator α_j. It is necessary to find the function φ. This dependence is universal. It does not depend from the concrete body and accelerator.

We have four known accelerations; it definite a 2×2 order matrix. These values are connected by some relation such that it is not dependent of bodies and accelerators. This property will be realized whenever it exists a nontrivial (i.e. not equal to zero) function Φ on the corresponding matrix set such as

$$\Phi(A) = 0, \qquad (1.4)$$

where A is known accelerators matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

The equality (1.4) is true for all pairs of bodies and accelerators.

Let M be the set of all bodies, and N be the set of all accelerators. Then the pair $\overline{\alpha}$, composed of the accelerators α_1,α_2 and designated by $\langle \alpha_1, \alpha_2 |$, is an element of the set $\mathfrak{N}^2$. The pair $\overline{i}$, composed of the bodies i_1,i_2 and designated by $| i_1, i_2 \rangle$, is an element of the set $\mathfrak{M}^2$. Then the equality (1.4), which definite the relation between chosen physical objects, can be transformed to

$$\Phi\left[\overline{\varphi}\left(\overline{\alpha},\overline{i}\right)\right] = 0 \ \forall \overline{\alpha} \in \mathfrak{N}^2,\ \overline{i} \in \mathfrak{M}^2, \qquad (1.5)$$

where

$$\overline{\varphi}\left(\overline{\alpha},\overline{i}\right) = \begin{pmatrix} \varphi(\alpha_1,i_1) & \varphi(\alpha_1,i_2) \\ \varphi(\alpha_2,i_1) & \varphi(\alpha_2,i_2) \end{pmatrix}.$$

The equality (1.5) can be interpreted as the specific equation with respect to the functions φ and Φ. It includes the considered equality (1.2), described the relation between measurable accelerations. But it was obtained with using the hypothesis of the existence a relation between pairs of bodies and accelerators without any a priori information about Newton law. Then we receive the following notion called the *phenomenological symmetry principle* [10].

Definition 1.1. *The equality* (1.5) *is called the equation of* ***mechanic movement physical structure***.

The general definition of the physical structure will be definite soon. Now note that the physical structure has two systematic parameters [10]. One of them is a *dimension*. It equals to (1,1) because of quantity of the considered physical objects parameters (the mass of the body and the force of the accelerator). The second parameter is a *rank*. It equals to (2,2) because of quantity of the physical objects (bodies and accelerators) in this equation. The substantiveness of this equation is defined by the following question.

Question 1.4. *What is a class of functions* φ *and* Φ, *satisfying the condition* (1.5)?

We can prove that the functional φ is defined by equality

$$\varphi(\alpha,i) = f\left[\xi^1(\alpha)x^1(i)\right] \qquad (1.6)$$

at enough natural assumptions, where functional ξ^1 and x^1, defined on the sets N and M consequently, and invertible function f are arbitrary. The value $\xi^1(\alpha)$ can be interpreted as the force of the accelerator α, and $x^1(i)$ can be interpreted as an inverse to the mass of the body i. Then we receive the equalities $F(\alpha) = \xi^1(\alpha),\ m(i) = 1/x^1(i).$

The matrix function Φ is defined by formula

$$\Phi\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = f^{-1}(a_{11})f^{-1}(a_{22}) - f^{-1}(a_{12})f^{-1}(a_{21}).$$

It is equal to

$$\Phi(A) = \left| f^{-1}\langle A \rangle \right|. \qquad (1.7)$$

The matrix $f^{-1}\langle A \rangle$ is determined by the formula

$$f^{-1}\langle A \rangle = \begin{pmatrix} f^{-1}(a_{11}) & f^{-1}(a_{12}) \\ f^{-1}(a_{21}) & f^{-1}(a_{22}) \end{pmatrix}.$$

The term in the right side of the equality (1.7) is its determinant. We can obtain the equality (1.2) now for the identical function f.

It is necessary to verify the influence of the arbitrariness for solutions of the equation (1.5) in its practical used, particularly in the determination of the force and the mass. Let us have two bodies and one accelerator. We get

$$\varphi(\alpha,i_k) = f\left[F(\alpha)/m(i_k)\right],\ k = 1,2.$$

By reversibility of the function *f* we have

$$f^{-1}\ \varphi(\alpha,i_k)\ = F(\alpha)/m(i_k),\ k=1,2.$$

After division of the first equality into the second one we find

$$\frac{m(i_2)}{m(i_1)}=\frac{f^{-1}\ \varphi(\alpha,i_1)}{f^{-1}\ \varphi(\alpha,i_2)}.$$

The values $f^{-1}\ \varphi(\alpha,i_1)$ and $f^{-1}\ \varphi(\alpha,i_2)$ can be interpreted as results of the measuring the relevant accelerations. The arbitrariness of the function *f* here implies the choice of the scale of the measuring. If the first body is chosen as the mass standard, then we can find the mass of the second body in this system of mass measuring. The force measuring can be realized analogically. Thus, the force and the mass can be defined on the basis of the equation of physical structure of mechanical movement unequivocally to within a choice of corresponding units of the measure.

Conclusion 1.5. *The equation of the mechanical movement physical structure is obtained with using only the existence of the universal law such that it restricts two arbitrary bodies and two arbitrary accelerators.*

The Newton law is a natural example of the physical structure for co-operating physical objects (bodies and accelerators here) with one characteristic (the mass for bodies and the force for accelerators). We determine physical structures with high dimensions (see [10]) whenever the objects have several characteristics.

2. Physical structure of electric circuits

Consider the electric circuit. After the connection of the conductor to the source of current (let it be a battery) we observe the current in the circuit. This process is described by *Ohm law for the complete circuit*

$$I=\frac{E}{R+\rho},$$

where I is a current strength, E is an electromotive force of the battery, ρ is its internal resistance, R is a resistance of the conductor.

In this case we have also two classes of physical objects (batteries and conductors). The current strength is a corollary of the interaction of these objects. It characterizes the considered process and can be measurable. However the electromotive force and the internal resistance are parameters of a battery, and the resistance is a property of a conductor. These parameters describe individual characteristics of considered physical objects.

Question 2.1. *Is it possible to verify the applicability of Ohm law for the complete circuit with using the measurable current strengths only*?

Transform Ohm law to

$$\frac{1}{I}=\frac{R+\rho}{E}=\frac{R}{E}+\frac{\rho}{E}.$$

We will use the following designation: $a=1/I,\ b^1=1/E,\ b^2=1/E.$ The battery α has two characteristics $b^1(\alpha)$ and $b^2(\alpha)$, and the conductor I has a unique characteristic $R(i)$.

Choose such quantity of physical objects for excluding all object characteristics from Ohm law. After the relevant transformation it includes only the measurable values of the current strengths. It is sufficient to have two batteries and three conductors for this aim. Then we obtain the equalities

$$b_j^1R_k+b_j^2=a_{jk},\ j=1,2;\ k=1,2,3; \qquad (2.1)$$

it is an analogues of the formulas (1.1).

Remark 2.1. The battery has the bigger characteristics than the conductor. Hence it is necessary to take more conductors than batteries for obtaining the corresponding equalities. In the opposite case we receive the large enough quantity of the unknown object characteristics for the given objects. We didn't this effect for Newton law because of equality between the quantities of the parameters for both physical objects classes.

Remark 2.2. We have six equations (2.1) with seven unknown values (one characteristic for each conductor, and two characteristics for each battery). However our aim is to exclude all objects characteristics from Ohm law, but not to receive the relations between the quantity of the equations and unknown values.

Exclude the unknown values from given formulas. Then

$$b_j^1\ \ R_1-R_k\ \ =a_{j1}-a_{jk},\ j=1,2;\ k=2,3.$$

We get the equality

$$\frac{a_{11}-a_{12}}{a_{11}-a_{13}}-\frac{a_{21}-a_{22}}{a_{21}-a_{23}}=0; \qquad (2.2)$$

it connects measurable parameters only as the condition (1.2).

Conclusion 2.1. *Two batteries and three conductors are sufficient for the experimental verification of Ohm law; then six measurable values of current strengths can be used for the verification of the equality* (2.2).

Conclusion 2.2. *Six current strengths, characterized the interactions between arbitrary pairs of batteries and conductors, are not unconditioned; one of them depends from others by equality* (2.2).

Conclusion 2.3. *It is not necessary to measure any electromotive forces and resistances for verification of Ohm law; we do not use these notions even.*

We have now the following question.

Question 2.2. *How we can find parameters of the battery and the conductor with using of measurable current strengths only?*

By last equality we obtain

$$\frac{E_2}{E_1}=\frac{b_1^1}{b_2^1}=\frac{a_{11}-a_{12}}{a_{21}-a_{22}}.$$

Choose the value E_1 as an electromotive force standard and determine by the experiment the corresponding values of the current strengths. Then we can find the comparative value of the second electromotive force, i.e. its value in comparison with given standard. Similarly choose the resistances R_1 and ρ_1 as standards. So the values of other resistances can be found by equalities

$$R_k+\rho_j=E_j a_{jk},\ j=1,2;\ k=1,2,3.$$

Conclusion 2.4. *The electromotive force and resistances can be determined by means of the measurable current strengths to within a choice of units.*

Remark 2.3. We had six equations (2.1) with seven unknown values. The five equations of them only are independent because of equality (2.2). After the choice of three standards we obtain four unknown values. Then one of the equations (2.1) can be excluded from the system; it connects the units with measurable values without any unknown parameters. So we receive five equations (one of them is depended from others) with four unknown values (two resistances of conductors and the electromotive force and the resistance for a battery). Hence there unknown values can be definite by standards and measurable values.

We supposed before that the standard form of the Ohm law is known.

Question 2.3. *Is it possible to restore the Ohm law with using of the connection type between batteries and conductors only?*

It is necessary to determine the law of the connection $\varphi=\varphi(\alpha,i)$ between the battery α and the conductor *i*, with using of the following assumption. Two batteries and three conductors are sufficient for obtaining the required law. Then we get the equalities

$$\varphi(\alpha_j,i_k)=a_{jk},\ j=1,2;\ k=1,2,3. \qquad (2.3)$$

It is an analogue of (1.3). The functional φ determines the value of the current strength for the given battery and conductor; it is unknown. Then six values of the current strengths need be connected by some equality. It can be described as the 2×3 order matrix. Besides it is not depend from the choice of physical objects. This property is realized whenever a nontrivial function Φ on the set of the corresponding matrixes exists such as

$$\Phi(A)=0. \qquad (2.4)$$

This equality is an analogue of (1.4). The matrix *A* is determine by the current strengths

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}.$$

The equality (2.4) is true for all pair of batteries and three conductors.

Let M be the set of all conductors, and N be the set of all batteries. Then the pair of batteries $\bar{\alpha}=\langle\alpha_1,\alpha_2|$ is an element of the set $\mathfrak{N}^2$, and the three conductors $\bar{i}=|i_1,i_2,i_3\rangle$ is an element of the set $\mathfrak{M}^3$. Then the equality (2.4) can be transformed to the form

$$\Phi\left[\bar{\varphi}\left(\bar{\alpha},\bar{i}\right)\right]=0\ \forall\bar{\alpha}\in\mathfrak{N}^2,\ \bar{i}\in\mathfrak{M}^3, \qquad (2.5)$$

where

$$\bar{\varphi}\left(\bar{\alpha},\bar{i}\right)=\begin{pmatrix}\varphi(\alpha_1,i_1) & \varphi(\alpha_1,i_2) & \varphi(\alpha_1,i_3)\\ \varphi(\alpha_2,i_1) & \varphi(\alpha_2,i_2) & \varphi(\alpha_2,i_3)\end{pmatrix}.$$

The equality (2.5) is the analogue of (1.5). It is a specific equation with respect to the maps φ and Φ. The considered equality (2.4) is its particular case.

Definition 2.1. *The equality* (2.5) *is the equation of the* ***electric circuit physical structure*****.**

This physical structure has a (2,1) dimension (we have two characteristics of a battery and one parameter of a conductor) and a (2,3) degree (two batteries and three conductors were chosen).

Our next problem is a describing of the solution set of the equation (2.5). We can prove that the function φ determines by the equality

$$\varphi(\alpha,i)=f\left[\xi^1(\alpha)x^1(i)+\xi^2(\alpha)\right], \qquad (2.6)$$

where the functionals ξ^1, ξ^2 on the set N, the functional x^1, and the function *f* are arbitrary. The value $x^1(i)$ can be interpreted here as a resistance of the conductor *i*, $\xi^1(\alpha)$ is an inverse value of the electromotive force of the battery α, and $\xi^2(\alpha)$ is an interior resistance of the battery α. Then we obtain the equalities

$$R(i)=x^1(i),\ E(\alpha)=1/\xi^1(\alpha),$$

$$r(\alpha)=\xi^2(\alpha)/\xi^1(\alpha).$$

The matrix function Φ is determined by formula

$$\Phi\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{pmatrix}=f^{-1}(a_{11})f^{-1}(a_{22})+f^{-1}(a_{12})f^{-1}(a_{23})+f^{-1}(a_{13})f^{-1}(a_{21})-$$
$$-f^{-1}(a_{11})f^{-1}(a_{23})-f^{-1}(a_{12})f^{-1}(a_{21})-f^{-1}(a_{13})f^{-1}(a_{22}).$$

This equality can by transform to

$$\Phi(A)=\left|f^{-1}\langle A\rangle\right|, \qquad (2.7)$$

where the matrix $f^{-1}\langle A\rangle$ is determined by

$$f^{-1}\langle A\rangle=\begin{pmatrix}1 & 1 & 1\\ f^{-1}(a_{11}) & f^{-1}(a_{12}) & f^{-1}(a_{13})\\ f^{-1}(a_{21}) & f^{-1}(a_{22}) & f^{-1}(a_{23})\end{pmatrix}.$$

The equality (2.2) follows from (2.7) for identical function *f*.

The parameters of a battery and a conductor can be found from the physical structure equation (2.5) by the described method.

Conclusion 2.5. *The equation of the* (2,3) *degree electric circuit physical structure is obtained with using only the universal law, which restricts two arbitrary batteries and three arbitrary conductors*; *it is a special form of Ohm law to within a choice of units of physical objects*.

We have two different example of the physical structure. So it is possible to determine a general physical structure.

3. Definition of physical structures

Determine hypothesizes and axioms of physical structures.

Hypothesis 3.1. *The physical structure characterizes a relation between two classes of objects*.

Axiom 3.1. *Two sets* N *and* M *are definite; its elements are called* ***physical objects***.

Thus the physical structure are definite on the pair of the objects classes (N,M). For example the mechanical movement physical structure is definite on the sets of accelerators N and bodies M, and the electric circuit physical structure is definite on the sets of batteries N and conductors M**.**

Hypothesis 3.2. *Every physical object has a finite quantity of characteristics; this quantity is constant for all objects of the given class.*

Axiom 3.2. *There are maps* $\xi:\mathfrak{N}\to\mathbb{R}^n$ *and* $x:\mathfrak{M}\to\mathbb{R}^m$, *called* ***characteristics*** *of physical* *objects; the pair of natural numbers* (n,m) *are called the* ***dimension*** *of the physical structure.*

It determines the set of characteristics $\xi(\alpha) = \xi^1(\alpha),...,\xi^n(\alpha)$ for all object $\alpha \in \mathfrak{N}$, and the set of characteristics $x(i) = x^1(i),...,x^m(i)$ for all object $i \in \mathfrak{M}$. For example, the accelerator is characterized by the force, and the body is characterized by the mass. Similarly the battery has an electromotive force and an interior resistance, and a conductor has a resistance. Then the dimension of the mechanical movement physical structure is equal to (1,1), and the electric circuit physical structure has a dimension (2,1).

Remark 3.1. The belonging of physical objects in the same class implies that these objects have same characteristics. Concrete characteristics of the given class have the physical dimensionality at least. So it is possible to add the analogical values.

Remark 3.2. The object can be included in different object classes. For example if it characterized by mass, it is a body and it can move. However this object with resistance is a conductor and it can to conduct the current. This situation is ordinary for mathematical objects. For example the real numbers set with addition is a commutative group. But this set with natural order is a linear ordering set.

Hypothesis 3.3. *There is a quantitative relation between characteristics of arbitrary two objects of different classes.*

Axiom 3.3. *There is a map* $\varphi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ *called* ***representator*** *of the physical structure.*

There is a function φ on the set $\mathbb{R}^n \times \mathbb{R}^m$ for an (n,m) order physical structure on the pair (N,M). This function describes a quantitative relation between characteristics of the both classes; it can be measured directly. Then the relation between the objects $\alpha \in \mathfrak{N}$ and $i \in \mathfrak{M}$ is described by the value $\langle \alpha | i \rangle = \varphi\ \xi(\alpha), x(i)$. Particularly there is a two arguments function. It describes the relation between characteristics of bodies and accelerators for the mechanic movement. Thus it can determine the acceleration (representator) $\varphi\ \xi(\alpha), x(i) = \xi^1(\alpha) / x^1(i)$ for the accelerator α with force $\xi^1(\alpha)$ and the body i with mass $x^1(i)$. The quantitative relation between an accelerator α and a body i is described by the term $\langle \alpha | i \rangle$. It is restricted by the Newton law. Analogically there is a two arguments function, which describes the quantitative relation between characteristics of batteries and conductors for the electric circuit. Particularly it can determine the current strength (representator) $\varphi\ \xi^1(\alpha), \xi^2(\alpha); x^1(i) = \xi^1(\alpha) / \left[x^1(i) + \xi^2(\alpha) \right]$ for a battery α with electromotive force $\xi^1(\alpha)$ and an interior resistance $\xi^2(\alpha)$ and a conductor i with resistance $x^1(i)$. Then the quantitative relation between a battery α and a conductor i is described by the term $\langle \alpha | i \rangle$. It is restricted by Ohm law.

Remark 3.3. The representator is a characteristic of the considered process. It describes the result of the interaction between physical objects of different classes. Values of the representator are measured as a rule by the practical experiment. Measurable characteristic are usually the result of the interaction but not an individual parameter of the interacting object.

Remark 3.4. The representator is a functional on the pair of physical objects. So we could exclude its characteristics of the consideration (see [10]).

Hypothesis 3.4. *There is a quantitative correlation between concrete numbers of objects of both classes; it depends only from quantitative relations between characteristics of every object pairs of the different classes.*

Axiom 3.4. (***Phenomenological symmetry principle*** [10]). *There is the pair of natural numbers* (r,s) *such as the function* Φ *that is determined on the* $r \times s$ *order matrix set* $\mathbb{R}^{r,s}$ *satisfies the* ***physical structure equation***

$$\Phi\ \overline{\varphi}\ \overline{\alpha}, \overline{i} = 0\ \forall \overline{\alpha} \in \mathfrak{N}^r, \overline{i} \in \mathfrak{M}^s, \quad (3.1)$$

where $\overline{\alpha} = \alpha_1,...,\alpha_r$, $\overline{i} = i_1,...,i_s$, $\overline{\varphi}(\overline{\alpha}, \overline{i})$ *is a matrix with elements* $\varphi\ \xi(\alpha_j), x(i_k)$. *The pair* (r,s) *is called the* ***rank*** *of the physical structure, and* Φ *is called its* ***verificator***.

Thus an ordered family $\overline{\alpha} \in \mathfrak{N}^r$ of the objects $\alpha_1,...,\alpha_r$ from the class N denoted by $\langle \alpha_1,...,\alpha_r |$, and an ordered family $\overline{i} \in \mathfrak{M}^m$ of the objects $i_1,...,i_s$ from the class M denoted by $| i_1,...,i_s \rangle$ are used for the description of (r,s) rank physical structure on the pair (N,M). Then the relation between object classes are determined by matrix $\bar{\varphi}(\bar{\alpha},\bar{i}) = \langle \alpha_1,...,\alpha_r | i_1,...,i_s \rangle$ with elements $\varphi\ \xi(\alpha_j), x(i_k)\ = \langle \alpha_j, i_k \rangle$. Particularly the pair of accelerators $\bar{\alpha} = \langle \alpha_1, \alpha_2 |$ and the pair of bodies $\bar{i} = \langle i_1, i_2 |$ determine the mechanic movement physical structure. Thus the connection between accelerators and bodies are described by 2×2 matrix $\langle \bar{\alpha}, \bar{i} \rangle = \langle \alpha_1, \alpha_2 | i_1, i_2 \rangle$ of relations $\langle \alpha_j, i_k \rangle$ between accelerator α_j and body i_k, where $j,k = 1,2$. Accordingly the pair of batteries $\bar{\alpha} = \langle \alpha_1, \alpha_2 |$ and three conductors $\bar{i} = \langle i_1, i_2, i_3 |$ determine the electric circuit physical structure. The relation between batteries and conductors are described by the matrix $\langle \bar{\alpha}, \bar{i} \rangle = \langle \alpha_1, \alpha_2 | i_1, i_2, i_3 \rangle$ of relations $\langle \alpha_j, i_k \rangle$ between the battery α_j and the conductor i_k, where $j = 1,2,\ k = 1,2,3$.

The term $\bar{\varphi}\ \bar{\alpha}, \bar{i}$ is a matrix with order $r\times s$. It can be interpreted as the point of Euclid space $\mathbb{R}^{rs}$. The set $\bar{\varphi}\ \mathfrak{N}^r, \mathfrak{M}^s$ of these points includes in a hypersurface of $\mathbb{R}^{rs}$, i.e. in a manifold of $rs-1$ dimension by Axiom 3.4. For example, there exists a function Φ for the mechanic movement physical structure that definite on a set of 2×2 order matrixes such as

$$\Phi\ \bar{\varphi}\ \bar{\alpha}, \bar{i}\ \ = 0\ \forall \bar{\alpha} \in \mathfrak{N}^2, \bar{i} \in \mathfrak{M}^2. \quad (3.2)$$

This function is definite by equality (1.7) and realizes the relation between four measurable accelerations. It does not depend from the choice of accelerators and bodies. Thus the set $\bar{\varphi}\ \mathfrak{N}^2, \mathfrak{M}^2$ of every four measurable accelerations is three dimensional. Similarly there exists a function Φ on the set of 2×3 order matrixes for the electric circuit physical structure such as

$$\Phi\ \bar{\varphi}\ \bar{\alpha}, \bar{i}\ \ = 0\ \forall \bar{\alpha} \in \mathfrak{N}^2, \bar{i} \in \mathfrak{M}^3. \quad (3.3)$$

This function is definite by the equality (2.7) and realizes the relation between six measurable current strengths. It does not depend from the choice of batteries and conductors. So the set $\bar{\varphi}\ \mathfrak{N}^2, \mathfrak{M}^3$ of six current strengths is five dimensional.

Remark 3.5. In really the set $\bar{\varphi}\ \mathfrak{N}^r, \mathfrak{M}^s$ can be a manifold with a dimension smaller than $rs-1$. It is possible an existence of several independent functions Φ that satisfy the equality (3.1).

The physical structure equation can be interpreted as the problem with respect to a representator and a verificator. The equalities (3.2) and (3.3) are equal to (1.5) and (2.5) for the considered examples.

Remark 3.6. Solvability of the physical structure equation testifies that the values of representators (measurable process characteristics) are not arbitrary for all families of physical objects. One of them at least depends from others by physical structure equation. This circumstance is just a sign of a physical law.

Axioms 3.1 – 3.4 can be called axioms of physical objects classes, characteristics and dimension, representators, rank and verificator consequently. It includes ten values $\mathfrak{N}, \mathfrak{M}, n, m, r, s, \xi, x, \varphi, \Phi$. Now we denote this ordering set the physical structure.

Definition 3.1. *Under Axioms* 3.1 – 3.4 $\mathfrak{N}, n, \xi, r\ ,\ \mathfrak{M}, m, x, s\ , \varphi, \Phi$ *is called the* ***physical structure*** *of the dimensional* (n,m) *and the rank* (r,s) *on the physical objects sets* (N,M), *where* ξ *and* x *are the characteristics,* φ *is the representator, and* Φ *is the verificator.*

Particularly the sets of accelerators with one characteristic (force) and bodies with one characteristic (mass) after the choice of two bodies and two accelerators with the representator determinate by the formula (1.6) (the accelerator is definite by Newton law), and the verificator determinate by the formula (1.7), form the mechanic movement physical structure. The sets of batteries with two characteristics (electromotive force and interior resistance) and conductors with one characteristic (resistance) after the choice of two batteries and three conductors with representator determinate by formula (2.6) (the current strength is definite the Ohm law), and the verificator determinate by formula (2.7), form the electric circuit physical structure.

4. Properties of physical structures

Note the symmetry of the physical objects classes. We deduce directly from the definition of the physical structure

Theorem 4.1.

If $\mathfrak{N},n,\xi,r$, $\mathfrak{M},m,x,s$ $,\varphi,\Phi$ *is the physical structure, then* $\mathfrak{M},m,x,s$, $\mathfrak{N},n,\xi,r$ $,\varphi^*,\Phi^*$ *is also the physical structure, where the maps* $\varphi^*:\mathbb{R}^m\times\mathbb{R}^n\to\mathbb{R}$ *and* $\Phi^*:\mathbb{R}^{s,r}\to\mathbb{R}$ *are definite by*

$$\varphi^*(y,\eta)=\varphi(\eta;y)\ \forall y\in\mathbb{R}^m,\eta\in\mathbb{R}^n,$$
$$\Phi^*(M)=\Phi(M^*)\ \forall M\in\mathbb{R}^{s,r};$$

besides M^* *is the transposed matrix M.*

Definition 4.1. *The physical structure* $\mathfrak{M},m,x,s$, $\mathfrak{N},n,\xi,r$ $,\varphi^*,\Phi^*$ *is called the* ***adjoint physical structure*** *with respect to* $\mathfrak{N},n,\xi,r$, $\mathfrak{M},m,x,s$ $,\varphi,\Phi$.

Objects classes and its characteristics are swapped for the adjoint structure. Each property of the initial structure has an analogue for the adjoint structure by the ***principle of duality***. The structure and adjoint one are mutually adjoint, i.e. the adjoint physical structure for the structure $\mathfrak{M},m,x,s$, $\mathfrak{N},n,\xi,r$ $,\varphi^*,\Phi^*$ is the initial structure $\mathfrak{N},n,\xi,r$, $\mathfrak{M},m,x,s$ $,\varphi,\Phi$.

Conclusion 4.1. *Every properties of the adjoint physical structure can be undeleted by the properties of the initial structure. So we can consider only the case* $m\geq n$.

Let us consider a map $\varphi:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}$, where $m\geq 1$, $n\geq 1$. We denote by $\langle\Phi,\varphi\rangle$ the composition of the map Φ and the transformation that takes the matrix with columns $\xi_1,...,\xi_r$ and $x_1,...,x_s$ to the matrix with general element $\varphi(\xi_j,x_k)$, $j=1,...,r$, $k=1,...,s$. Determine the modified concept of the representator and the verificator.

Definition 4.2. *The maps* $\varphi:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}$ *and* $\Phi:\mathbb{R}^{r,s}\to\mathbb{R}$ *are called the* ***representator*** *and the* ***verificator*** *of a physical structure of dimension* (n,m) *and rank* (r,s) *whenever*

$$\langle\Phi,\varphi\rangle=0. \qquad (4.1)$$

Thus the value of the function Φ in the matrix with general term $\varphi(\xi_j,x_k)$ is equal to zero for all $\xi_j\in\mathbb{R}^n$, $x_k\in\mathbb{R}^m$, where $j=1,...,r$, $k=1,...,s.$ These notions coincide with the considered ones if the physical objects classes are the corresponding Euclid spaces. We will consider the verificator and the representator in the sense of Definition 4.2.

Particularly the two arguments function

$$\varphi(\xi_1,x_1)=f\ \ \eta^1(\xi_1)y^1(x_1) \qquad (4.2)$$

is the analogue of (1.6), and the corresponding matrix function Φ definite by (1.7). These functions are the verificator and the representator of the (2,2) rank physical structure for all functions η^1, y^1 and all reversible function *f*. The three arguments function

$$\varphi\ \ \xi_1,\xi_2;x_1\ \ =f\ \ \eta^1(\xi_1,\xi_2)y^1(x_1)+\eta^2(\xi_1,\xi_2) \qquad (4.3)$$

is the analogue of (2.6), and the function Φ, which is definite by (2.7). These functions are the verificator and the representator of the (2,3) rank physical structure for all functions η^1, η^2, y^1 and all reversible function *f*.

Remark 4.1. Physical objects sets are not use in Definition 4.1 because the physical structure equation connects numerical characteristics of objects but not objects directly. For example we can determine the *electric subcircuit physical structure* by means of the Ohm law $I=E/R.$ It has also the rank (2,2); so the two arguments function *φ*, which was definite before, and the corresponding function Φ are its verificator and representator.

Theorem 4.2. *The ten* $\mathfrak{N},n,\xi,r$, $\mathfrak{M},m,x,s$ $,\varphi,\Phi$ *is the physical structure of the dimension* (n,m) *and the rank* (r,s) *for all sets* M,N, *maps* $\xi:\mathfrak{N}\to\mathbb{R}^n$, $x:\mathfrak{M}\to\mathbb{R}^m$, *the representator* φ *and the verificator* Φ.

The value of the function Φ in a matrix with general term $\varphi(\xi_j,x_k)$ equals to zero indeed by

Definition 4.2 for all $\xi_j \in \mathbb{R}^n$, $x_k \in \mathbb{R}^m$, where $j=1,...,r$, $k=1,...,s$. Determine $\xi_j = \xi(\alpha_j)$, $x_k = x(i_k)$. Then the value of the function Φ in the matrix with general term $\varphi(\xi_j, x_k)$ equals to zero, i.e. the condition (4.1) is true.

Conclusion 4.2. *The physical structure equation in the form* (4.1) *connects only its representator and verificator; these notions are adjoint in some sense.*

Every maps $\xi : \mathfrak{N} \to \mathbb{R}^n$, $x : \mathfrak{M} \to \mathbb{R}^m$ can be chosen as characteristics of (n,m) dimension structure on (M,N) by Theorem 4.2. Different characteristics signify different scales of measurement of physical objects. Particularly arbitrariness of the functionals ξ^1 and x^1 in the formula (1.6) signify the freedom of the choice of the measuring scales for forces and masses; and the arbitrariness of the functionals ξ^1, ξ^2 and x^1 in the equality (2.6) signify the freedom of the choice of the measuring scales for an electromotive force and the interior resistance of a battery and the resistance of a conductor. So the different characteristics determine the same structure in this case.

Remark 4.2. There are two types of parameters in the definition of a physical structure. The rank, dimension, representator, and verificator are the mathematical notions per se. However the physical objects classes and its characteristics have a physical sense. It uses for the interpretation of the results of an analysis of the physical structure equation, but not for this analysis directly.

Theorem 4.3. *If the pair* (φ,Φ) *is a representator and a verificator of a physical structure of dimension* (n,m) *and rank* (r,s), *then for all invertible function f and all maps* $\mathrm{H}:\mathbb{R}^n \to \mathbb{R}^n$ *and* $\Upsilon:\mathbb{R}^m \to \mathbb{R}^m$ *the pair* $(\varphi_f^{\mathrm{H}\Upsilon}, \Phi_f)$ *has the analogical properties. The transformation* $\varphi_f^{\mathrm{H}\Upsilon}$ *takes vectors* $\xi \in \mathbb{R}^n$ *and* $x \in \mathbb{R}^m$ *to the number* $f\ \varphi\ \mathrm{H}\xi, \Upsilon x$ *there, and the map* $\Phi_f : \mathbb{R}^{r,s} \to \mathbb{R}$ *takes the matrix with the general term* a_j^k *to the value of the function* Φ *in the matrix with the general term* $f^{-1}\ a_j^k$.

Proof. Let a pair (φ,Φ) satisfies the equality (4.1). Then the value of the function Φ in a matrix with the general term $\varphi(\xi_j, x_k)$ equals to zero for all $\xi_j \in \mathbb{R}^n$, $x_k \in \mathbb{R}^m$, where $j=1,...,r$, $k=1,...,s$. Determine the mapping from $\mathbb{R}^{r,n} \times \mathbb{R}^{s,m}$ to $\mathbb{R}^{r,s}$ for $\varphi_f^{\mathrm{H}\Upsilon}$ that takes a matrixes with columns $\xi_1,...,\xi_r$ and $x_1,...,x_s$ to the matrix with the general term $f\ \varphi\ \mathrm{H}\xi_j, \Upsilon x_k$. The value Φ_f of this matrix equals to the value of the function Φ in a matrix with general term $\varphi\ \xi'_j, x'_k$, where $\xi'_j = \mathrm{H}\xi_j$, $x'_k = \Upsilon x_k$. By conditions $\xi'_j \in \mathbb{R}^n$, $x'_k \in \mathbb{R}^m$, we obtain that the value of Φ_f in a matrix with general term $f\ \varphi\ \mathrm{H}\xi_j, \Upsilon x_k$ equals to zero for all $\xi_j \in \mathbb{R}^n$, $x_k \in \mathbb{R}^m$. Then we get $\left\langle \Phi_f, \varphi_f^{\mathrm{H}\Upsilon} \right\rangle = 0$. This completes the proof of the theorem.

Thus there is a class of representators and verificators of the physical structure. Now we introduce the following concept.

Definition 4.3. *Two pair representator-verificator* (φ,Φ) *and* (ψ,Ψ) *of a physical structure with dimension* (n,m) *and the same rank are* ***equivalent*** *whenever there exists an invertible function f and invertible maps* $\mathrm{H}:\mathbb{R}^n \to \mathbb{R}^n$ *and* $\Upsilon:\mathbb{R}^m \to \mathbb{R}^m$ *such as* $\psi = \varphi_f^{\mathrm{H}\Upsilon}$, $\Psi = \Phi_f$. *The corresponding class of equivalence is called the* ***physical structure generator*** *of the given dimension and rank.*

Concrete representators determine the scales of measurement for the corresponding values (accelerations for mechanical movement, current strengths for the electric circuit). It is natural to identify physical structures with same physical objects sets and ranks if its representators and verificators are equivalent. For example the formulas (4.2), (1.7) and (4.3), (2.7) determine representators and verificators of mechanical movement structure and electric circuit structure for all function *f*; so it characterizes a physical structure generator.

Conclusion 4.3. *The physical structure of the concrete dimension and rank are determined by the physical structure generator, but not by a concrete representator and verificator*.

With using this result we conclude that the description of physical structures adds up to find its generator.

Definition 4.4. *Physical structures are* ***isomorphic*** *if it has the same rank, dimension, and generator.*

Isomorphic physical structures have same mathematical properties. It can differ with physical interpretation (classes of physical objects and their characteristics) or equivalent representator and verificator. For example, the physical structures of the rectilinear mechanic movement and the electric subcircuit (the battery is characterized only by the electromotive force but not by the interior resistance) are isomorphic.

Conclusion 4.4. *Physical structures are classified accurate within isomorphism.*

Remark 4.3. We can use the inverse value of the mass for analysis of Newton law. We transform variables for analysis of Ohm law. However the corresponding physical law and its physical structure do not change. Thus we have a possibility to change physical characteristics but not only its scales of measurement. The obtaining physical structures are isomorphic; so it has the same sense.

We can determine correlations between rank and dimension of the structure.

Theorem 4.4. *If the pair* (φ,Φ) *is the representator and the verificator of a physical structure with dimension* (n,m) *and rank* (r,s), *then maps* $\Phi':\mathbb{R}^{r,s'}\to\mathbb{R}$ *and* $\varphi':\mathbb{R}^{n'}\times\mathbb{R}^{m}\to\mathbb{R}$ *for all natural numbers* $s'>s$ *and* $n'<n$ *exists such that* (φ,Φ') *is the representator and the verificator of a physical structure with dimension* (n,m) *and rank* (r,s'), *and* (φ,Φ') *is the representator and the verificator of a physical structure of dimension* (n',m) *and rank* (r,s).

Proof. It is sufficient to consider the cases $s'=s+1$ and $n'=n-1$. Suppose the existence of maps $\varphi:\mathbb{R}^{n}\times\mathbb{R}^{m}\to\mathbb{R}$ and $\Phi:\mathbb{R}^{r,s}\to\mathbb{R}$ that satisfy the equality (4.1), i.e.

$$\Phi\begin{pmatrix}\varphi(\xi_1,x_1)\ \dots\ \varphi(\xi_1,x_{1s})\\ \dots\quad\dots\quad\dots\\ \varphi(\xi_{1r},x_1)\ \dots\ \varphi(\xi_{1r},x_{1s})\end{pmatrix}=0\ \forall\xi_j\in\mathbb{R}^{n},x_k\in\mathbb{R}^{m},\ j=1,\dots,r,\ k=1,\dots,s. \qquad (4.4)$$

Determine the matrix function $\Phi':\mathbb{R}^{r,s+1}\to\mathbb{R}$ by equality

$$\Phi'\begin{pmatrix}a_{11}\ \dots\ a_{1s}\ a_{1s+1}\\ \dots\quad\dots\\ a_{r1}\ \dots\ a_{rs}\ a_{rs+1}\end{pmatrix}=\Phi\begin{pmatrix}a_{11}\ \dots\ a_{1s}\\ \dots\quad\dots\\ a_{r1}\ \dots\ a_{rs}\end{pmatrix}+\Phi\begin{pmatrix}a_{12}\ \dots\ a_{1s+1}\\ \dots\quad\dots\\ a_{r2}\ \dots\ a_{rs+1}\end{pmatrix}+\dots+\Phi\begin{pmatrix}a_{1s+1}\ a_{11}\ \dots\ a_{1s-1}\\ \dots\quad\dots\\ a_{rs+1}\ a_{r1}\ \dots\ a_{rs-1}\end{pmatrix}.$$

we get

$$\Phi'\begin{pmatrix}\varphi(\xi_1,x_1)\ \dots\ \varphi(\xi_1,x_s)\ \varphi(\xi_1,x_{s+1})\\ \dots\quad\dots\quad\dots\\ \varphi(\xi_r,x_1)\ \dots\ \varphi(\xi_r,x_s)\ \varphi(\xi_r,x_{s+1})\end{pmatrix}=\Phi\begin{pmatrix}\varphi(\xi_1,x_1)\ \dots\ \varphi(\xi_1,x_s)\\ \dots\quad\dots\quad\dots\\ \varphi(\xi_r,x_1)\ \dots\ \varphi(\xi_r,x_s)\end{pmatrix}+$$

$$+\dots+\Phi\begin{pmatrix}\varphi(\xi_1,x_{s+1})\ \dots\ \varphi(\xi_1,x_{s-1})\\ \dots\quad\dots\quad\dots\\ \varphi(\xi_r,x_{s+1})\ \dots\ \varphi(\xi_r,x_{s-1})\end{pmatrix}=0\ \forall\xi_j\in\mathbb{R}^{n},x_k\in\mathbb{R}^{m}$$

$, j=1,\dots,r,\ k=1,\dots,s+1$

by (4.4). Then the pair (φ,Φ') is in really the representator and the verificator of the corresponding physical structure.

Determine now the map $\varphi':\mathbb{R}^{n-1}\times\mathbb{R}^m\to\mathbb{R}$ by formula

$$\varphi'(\xi;x)=\varphi(\xi,\xi^*;x)\ \forall\xi\in\mathbb{R}^{n-1},x\in\mathbb{R}^m,$$

where ξ^* is an arbitrary real number. Then we obtain the equality

$$\Phi\begin{pmatrix}\varphi'(\xi_1,x_1)\ \dots\ \varphi'(\xi_1,x_s)\\ \dots\quad\dots\quad\dots\\ \varphi'(\xi_r,x_1)\ \dots\ \varphi'(\xi_r,x_s)\end{pmatrix}=\begin{pmatrix}\varphi(\xi_1,\xi^*;x_1)\ \dots\ \varphi(\xi_1,\xi^*;x_s)\\ \dots\quad\dots\quad\dots\\ \varphi(\xi_r,\xi^*;x_1)\ \dots\ \varphi(\xi_r,\xi^*;x_s)\end{pmatrix}=0\ \forall\xi_j\in\mathbb{R}^{n-1},x_k\in\mathbb{R}^m$$

for all $j=1,...,r,\ k=1,...,s$ by (4.4), i.e. (φ',Φ) is the representator and the verificator of a corresponding physical structure with rank (r,s) and dimension (n',m). This completes the proof of the theorem.

We deduce from Theorem 4.4 and principle of duality that if (φ,Φ) is a representator and a verificator of a physical structure with dimension (n,m) and rank (r,s), then for all $r'>r$ it exist the matrix function Φ'' such as (φ,Φ'') is the representator and the verificator of a physical structure with same dimension and rank (r',s); and for all natural number $m'<m$ it exist the function φ'' such as (φ'',Φ) is the representator and the verificator of a physical structure with same rank and dimension (n,m').

Conclusion 4.5. *For any physical structure it exists a physical structure with greater rank and same dimension and representator, and a physical structure with lesser dimension and same rank, if the initial dimension is not equal to* (1,1).

We conclude by obtaining result:

Conclusion 4.6. *The dimension and the rank* (*the representator and the verificator too*) *are the dual notions.*

Remark 4.4. The representator is a function of objects characteristics. So it connects to the dimension of a physical structure, because it equals to the number of characteristics of objects classes. Contrariwise the verificator is definite on a matrix set; its order equals to numbers of chosen objects. It conforms to the rank of the physical structure.

Physical structures of greater ranks and same dimensions of smaller dimensions and same ranks, obtained by means of Theorem 4.4 are trivial. It does evidently any practical interest. In really we make more experiments for the (r,s') rank structure than for the structure with rank (r,s) whenever $s'>s$ without any additional information. An existence of physical structures of same rank and smaller dimension indicates about surplus information by Theorem 4.4, because one of the given physical characteristics at least can be excluded from the determination of the physical law. Then the following classes of physical structures generators are of interest.

Definition 4.4. *The generator of a physical structure of dimension* (n,m) *and rank* (r,s) *are called* ***nontrivial*** *if there is no generator of a physical structure of same dimension and rank* (r',s') *such as* $r'\le r$ *and* $s\le s'$ (*one of these inequality at least is strong*); *and is no generator of a physical structure of same rank and dimension* (r',s') *such as* $m\le m'$ *and* $n\le n'$ (*one of these inequality at least is strong*).

Conclusion 4.7. *Physical structures with nontrivial generators are only of interest.*

We considered before the mechanical movement physical structure of dimension (1,1) and rank (2,2) and the electric circuit physical structure of dimension (2,1) and rank (2,3). The dimension (n,m) and rank (r,s) satisfy the equalities $m=r-1,\ n=s-1$ in both cases. It is interesting that these conditions are realized for all practical examples of the physical structures described by Kulakov (see [10]).

Definition 4.5. *The physical structure with dimension* (n,m) *and rank* (r,s) *is called* ***Kulakov structure,*** *if* $m=r-1,\ n=s-1$.

Remark 4.5. There is sufficient to use only dimension or rank of Kulakov structure because of the existence of the connection between these parameters. For example, Kulakov structures are isomorphic whenever its ranks and generator are equivalent.

Remark 4.6. The connection between the rank and the dimension of Kulakov structures is an additional corroboration of the duality between these parameters. Similarly the condition (4.1) characterizes the relation between the representator and the verificator.

There is an important class of generators of Kulakov structures with complete description and serious applications (see [10]).

Definition 4.6. *The generator of Kulakov structure of rank* $(m+1, n+1)$ *is said to be* ***Mikhaylitchenko generator*** *if the corresponding functions* φ *and* Φ *are smooth enough and nondegenerate in the following sense. The maps* $\varphi^m : \mathbb{R}^m \to \mathbb{R}^m$ *and* $\varphi_n : \mathbb{R}^n \to \mathbb{R}^n$, *characterized by the equalities*

$$\varphi^m(x) = \varphi(\xi_1, x), ..., \varphi(\xi_m, x) \ ,$$

$\varphi_n(\xi) = \varphi(\xi, x_1), ..., \varphi(\xi, x_n)$ *, have the ranks m and n accordingly for all vectors* $\xi_1, ..., \xi_m \in \mathbb{R}^n$, $x_1, ..., x_n \in \mathbb{R}^m$, *and one of the partial derivatives at least of* Φ *is not equal to zero.*

By Mikhaylitchenko Theorem (see [11], and [22], p. 18) we get

Theorem 4.5. *There exist a unique Mikhaylitchenko generator for all Kulakov structure of rank* (2,2); *it is definite by functions*

$$\varphi(\xi_1, x_1) = \xi_1 x_1, \qquad \Phi\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

There exist exactly two Mikhaylitchenko generators for all Kulakov structure of rank (*r*,*r*), $r > 2$; *it is definite by functions*

$$\varphi(\xi_1, ..., \xi_{r-1}; x_1, ... x_{r-1}) = \sum_{j=1}^{r-1} \xi_j x_j \ ,$$

$$\Phi\begin{pmatrix} a_{11} & ... & a_{1r} \\ ... & & ... \\ a_{r1} & ... & a_{rr} \end{pmatrix} = \begin{vmatrix} a_{11} & ... & a_{1r} \\ ... & & ... \\ a_{r1} & ... & a_{rr} \end{vmatrix};$$

$$\varphi(\xi_1, ..., \xi_{r-1}; x_1, ... x_{r-1}) = \sum_{j=1}^{r-2} \xi_j x_j + \xi_{r-1} + x_{r-1}$$

$$\Phi\begin{pmatrix} a_{11} & ... & a_{1r} \\ ... & & ... \\ a_{r1} & ... & a_{rr} \end{pmatrix} = \begin{vmatrix} 0 & 1 & ... & 1 \\ 1 & a_{11} & ... & a_{1r} \\ ... & ... & & ... \\ 1 & a_{r1} & ... & a_{rr} \end{vmatrix}.$$

There exist a unique Mikhaylitchenko generator for all Kulakov structure of rank $(r, r+1)$, $r \geq 2$; *it is definite by functions*

$$\varphi(\xi_1, ..., \xi_r; x_1, ... x_{r-1}) = \sum_{j=1}^{r-1} \xi_j x_j + \xi_r \ ,$$

$$\Phi\begin{pmatrix} a_{11} & ... & a_{1r+1} \\ ... & & ... \\ a_{r1} & ... & a_{rr+1} \end{pmatrix} = \begin{vmatrix} 1 & ... & 1 \\ a_{11} & ... & a_{1r+1} \\ ... & ... & ... \\ a_{r1} & ... & a_{rr+1} \end{vmatrix}.$$

There exist a unique Mikhaylitchenko generator for all Kulakov structure of rank (2,4); *it is definite by functions*

$$\varphi(\xi_1, \xi_2, \xi_3; x_1) = \frac{\xi_1 x_1 + \xi_2}{\xi_3 + x_1},$$

$$\Phi\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{11}a_{21} & a_{12}a_{22} & a_{13}a_{23} & a_{14}a_{24} \end{vmatrix}$$

Mikhaylitchenko generators for all Kulakov structures of rank (r, s) *with* $s > r$ *do not exist.*

Remark 4.7. Mikhaylitchenko generators are definite by elemental functions of the corresponding equivalence classes.

Remark 4.8. The existence of the (2,2) rank physical structure are proved by Kulakov [2].

The characterization of Mikhaylitchenko generators for structures of rank (r, s) for $r > s$ is corollary of the Theorem 4.5 and duality principle.

Conclusion 4.8. Theorem 4.5 assures the description of generators and verificators of the arbitrary ranks Kulakov structures with assumption of smoothness and nondegeneracy.

Conclusion 4.9. There are unique accurate within of isomorphism Kulakov structures of ranks $(r, r+1)$ and (2,4) and exactly two structures of ranks (*r*,*r*). There not exist other Kulakov structures of ranks (r,s) for $s > r$.

Conclusion 4.10. Mikhaylitchenko generators are nontrivial generators of Kulakov structures.

Remark 4.9. Existence of many generators of physical structure signifies that there are many non-isomorphic structures of the given rank and the dimension. Then it is possible many admissible physical laws of the given type. It seems that we do not prefer one of them to another for the given phenomenon. There is an analogy here to a situation when the mathematical model of process has the nonunique solution. For example consider the Euler core problem. It is possible to calculate a profile, which will accept vertically located core by a force in top end. However we do not any possibility to divine its deviation in the right or the left side because of equality of the corresponding solutions of the considered boundary problem.

Remark 4.10. The concrete number of physical structure generators seems its important parameter. The existence of the generator and solvability of the physical structure equation testifies that the values of the characteristics of various interactions for the arbitrary chosen physical objects are not arbitrary. One of them at least depends from others by the physical structure equation.

Remark 4.11. We could ask, what is the cause of the assertions of Theorem 4.6 and Mikhaylitchenko Theorem? Let (r,s) be the rank of a structure. Then we choose *r* objects of the first type and *s* objects of the second one. There are *rs* relations between them; so we obtain *rs* equations. We do not know $s-1$ characteristics for *r* objects of the first type and $r-1$ characteristics for *s* objects of the second one. So we have $r(s-1)+s(r-1)$ unknown values. The quantity of unknowns is greater in generally than the quantity of equations. There is the question: under what conditions the ratio of quantity of unknown values to number of equations will be minimum, i.e. where occurrence of solutions most likely? Let the sum $q = r + s$ be fixed. Definite the function $f = f(r)$ that is the ratio of quantity of unknown values to number of equations for $s = q - r$, i.e. $f(r) = 2 - 1/r + 1/(r-q)$. It derivative is $f'(r) = q(q-2r)/[r(r-q)]^2$. After equalization to zero we obtain $r = q/2$, so $r = s$. It is the point of minimum obviously. Then the ratio of the quantity of unknown values to number of equations is minimal on the diagonal just, i.e. for $r = s$. It is clear that the relation between quantities of equations and unknowns are difficult enough. The equations are not independent because of the physical structure equations and existence of units. However it is not random that the system underdetermination (the existence of two non-isomorphic solutions) is realized exactly for the minimal ratio of the quantity of unknowns (large relatively in generally) to the quantity of equations (large in generally). We get a unique solution near the diagonal, where the balance of the quantity of unknowns to the number of equation is small enough. But we obtain the system overdetermination far off diagonal where the balance of the quantity of unknowns to the number of equation is large enough. The solution does not exist in this situation.

Remark 4.12. The conditions of smoothness and nondegeneracy are principled. Particularly we can determine degenerate structure generator for the (2,2) order Kulakov structure by formulas

$$\varphi(\xi, x) = \xi, \ \Phi\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

and the corresponding nonsmooth degenerate by equalities

$$\varphi(\xi, x) = \max \ \xi, x \ ,$$

$$\Phi\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \max \ a_{11}, a_{22} \ - \max \ a_{12}, a_{21} \ .$$

Remark 4.13. We do not know haw many exists nontrivial generator of Kulakov structures, which are not Mikhaylitchenko generators. We do not know also whether they have any physical sense.

Remark 4.14. It seems that physical structures, which are not Kulakov structures, are not researched. We do not know even whether they have any nontrivial generators.

5. Addition

Determine physical structures with coincident objects classes. The dimension (quantity of object characteristics) and the rank (quantity of chosen objects) are described by natural numbers but not by pairs for this case.

Definition 5.1. *Consider a set* N, *natural numbers n and r, maps* $\xi:\mathfrak{N}\to\mathbb{R}^n$, $\varphi:\mathbb{R}^{2n}\to\mathbb{R}^n$, *and a function of* $r\times r$ *order matrix such that*

$$\Phi\ \overline{\varphi}(\overline{\alpha})\ =0\ \forall\,\overline{\alpha}\in\mathfrak{N}^r, \tag{5.1}$$

where $\overline{\alpha}=\ \alpha_1,...,\alpha_r$, *and* $\overline{\varphi}(\overline{\alpha})$ *is the matrix with elements* $\varphi\ \xi(\alpha_j),\xi(\alpha_k)$, $j,k=1,...,r$. *Then the six* $\mathfrak{N},n,\xi,r,\varphi,\Phi$ *is called the* ***physical structure*** *on the set* N *of the dimension n, the rank r, the characteristics* ξ, *the representator* φ, *and the verificator* Φ.

Consider the interesting example of this structure given by Kulakov [10]. Determine the Euclid plane $\mathfrak{N}=\mathbb{R}^2$. For all point α (physical object) two Descartes coordinates (characteristics) exist denoted by $\xi(\alpha)=\ \xi^1(\alpha),\xi^2(\alpha)$; so $n=2$. We choose the distance between points as a representator (measurable parameter). Then we have

$$\varphi\ \xi(\alpha_1),\xi(\alpha_2)\ =\sqrt{\left[\xi^1(\alpha_1)-\xi^1(\alpha_2)\right]^2+\left[\xi^2(\alpha_1)-\xi^2(\alpha_2)\right]^2}$$

It is known Tartaglia formula; it determines the volume of the tetrahedron by means of its sides

$$V^2=\frac{(-1)^3}{2^3(3!)^2}\begin{vmatrix}0 & 1 & 1 & 1 & 1\\ -1 & a_{11} & a_{12} & a_{13} & a_{14}\\ -1 & a_{21} & a_{22} & a_{23} & a_{24}\\ -1 & a_{31} & a_{32} & a_{33} & a_{34}\\ -1 & a_{41} & a_{42} & a_{43} & a_{44}\end{vmatrix},$$

where a_{jk} is the square of the distance between j-th top and k-th top of the tetrahedron. It is obvious that this volume is equal to zero whenever four considered points are situated in the same plane. Thus we obtain the equality (5.1) for $r=4$ and the function Φ of four order square matrix with elements a_{jk}, determined by the determinant in the right side of the last equality. Then we receive *metric physical structure of Euclid plane* with dimension 2, the rank 4, the characteristics "Descartes coordinates", the representator "distance", and the verificator, determined by Tartaglia formula.

The metric structure of Euclid plane permits to make experimental verification of the concrete physical law as the physical structures of the mechanical movement and the electric circuit.

Remark 5.1. After the arbitrary choice of points on the plane we do not obtain the arbitrary values of the distances between these points. One of them depends from others. We had a similar situation in the previous examples. This circumstance permits to interpret this relation as a physical structure equation and to classify the metric relations on Euclid plane and the notion of Definition 5.1 to the category of physical structures.

Analogical results can be obtained for the spaces with arbitrary dimension (see Kulakov [10], Chapter 7).

Remark 5.2. Mikhaylitchenko interprets the representator as a generalized metric because it has relations with metric in the considered example and its generalizations. So the physical structures with the representator that has some independent components (we will consider it later) were be called *polymetric* (see [21] and [23] for the case of one objects set, and [19], [20] for the case of two objects sets).

Next step of physical structure generalization conforms to the situation of many characteristics of interactions between physical objects. Then values of representators are vectors but not numbers. The natural example of this situation is given by the second Newton law for the plane movement.

Consider the set of bodies M and the set of accelerators N. The body i has the unique characteristic (mass), but the accelerator α is characterized by the force vector with components $F^1=\xi^1(\alpha)$ and $F^2=\xi^2(\alpha)$. The corresponding movement is described by the acceleration vector

$(a^1, a^2) = \varphi(\alpha, i) = \left(\varphi^1(\alpha, i), \varphi^2(\alpha, i)\right)$. Using Newton law for the plane movement we obtain the relation between accelerations mass and forces $a^l = F^l / m,\ l = 1,2$. After the choice of two bodies i_1, i_1 and two accelerators α_1, α_2 we obtain the equalities $a_{jk}^l = F_k^l / m_j,\ j,k = 1,2,\ l = 1,2,$ where $m_j = x(i_j),\quad F_k^l = \xi^l(\alpha_k),\quad a_{jk}^l = \varphi^l(\alpha_k, i_j),$ $j,k,l = 1,2.$ Exclude forces and masses from it. We get the equalities $a_{11}^l a_{22}^l - a_{12}^l a_{21}^l = 0,\ l = 1,2;$ it is analogue to (1.2). Thus it exist the maps $\varphi : \mathfrak{N} \times \mathfrak{M} \to \mathbb{R}^2$ and $\Phi : \mathbb{R}^{2,2,2} \to \mathbb{R}^2$, where $\mathbb{R}^{2,2,2}$ is the set of two order cubic matrixes (three dimensional $2\times2\times2$ order matrixes) such as

$$\Phi\left(\bar{\varphi}\left(\bar{\alpha}, \bar{i}\right)\right) = 0\ \forall \bar{\alpha} \in \mathfrak{N}^2, \bar{i} \in \mathfrak{M}^2, \qquad (5.2)$$

where $\bar{\alpha} = \langle \alpha_1, \alpha_2 |,\ \bar{i} = | i_1, i_2 \rangle,\ \bar{\varphi}\left(\bar{\alpha}, \bar{i}\right)$ is the two order cubic matrixes with components $\varphi^l(\alpha_k, i_j)$. Particularly we have the equalities

$$\varphi^l(\alpha, i) = \xi^l(\alpha) / x(i),$$

$$\Phi(A) = \left(\Phi^1(A), \Phi^2(A)\right);\ \Phi^l(A) = \begin{vmatrix} a_{11}^l & a_{12}^l \\ a_{21}^l & a_{22}^l \end{vmatrix},\ l = 1,2,$$

where A is a two order cubic matrix with general element a_{jk}^l. The equality (5.2) is a two dimensional vector analogue of the equation (3.2) of the one dimensional mechanical movement physical structure. So we have the plane mechanical movement physical structure.

Remark 5.3. The representator of this structure is degenerate. Thus physical structures with degenerate representators can have some physical application, although majority of these structures do not have apparently any practical sense. The physical structure of one dimensional movement with rotation is considered in [25]. The bodies set and accelerators set are two-dimensional in this case. The corresponding two-component representator is not degenerate.

Thus there is two-dimensional (according to number of process characteristics, i.e. to dimension of set representator values) physical structure of the dimension (2,1) (the accelerator has two characteristics, and the body has a unique characteristic), the rank (2,2) (two objects from both classes are chosen), and two orders (according to quantities of independent equalities that connect measurable parameters, i.e. to the dimension of set verificator values) on the sets of bodies and accelerators with characteristics "mass of the body" and "components of vector force", the representator "components of the vector acceleration", and the verificator, which was determinate before.

Determinate also structures with arbitrary quantity of objects classes and arbitrary connections between considered objects.

Definition 5.2. *Consider sets* $\mathfrak{M}_1, \ldots, \mathfrak{M}_p$, *natural numbers* $m_1, \ldots, m_p$ $r_1, \ldots, r_p$, q, s, *and maps* $\xi_i : \mathfrak{M} \to \mathbb{R}^{m_i}$, $i = 1, \ldots, p$, $\varphi : \prod_{i=1}^{p} \mathbb{R}^{m_i} \to \mathbb{R}^s$; *and* Φ *maps the p-dimensional* $r_1 \times \ldots \times r_p$ *order matrix to q-dimensional vector such as*

$$\Phi\left(\bar{\varphi}(\bar{\alpha})\right) = 0\ \forall \bar{\alpha} \in \prod_{i=1}^{p} \mathfrak{M}_i^{r_i}, \qquad (5.3)$$

where $\bar{\alpha} = \left(\bar{\alpha}^1, \ldots, \bar{\alpha}^p\right)$, $\bar{\alpha}^i \in \mathfrak{M}_i^{r_i}$, $\bar{\varphi}(\bar{\alpha})$ *is an ordering set of s p-dimensional matrixes with components* $\varphi^1\left(\xi_1(\bar{\alpha}_{j_1}^1); \ldots; \xi_p(\bar{\alpha}_{j_p}^p)\right)$, …, $\varphi^s\left(\xi_1(\bar{\alpha}_{j_1}^1); \ldots; \xi_p(\bar{\alpha}_{j_p}^p)\right)$, $j_i = 1, \ldots, r_i$, $i = 1, \ldots, p$, $\varphi = (\varphi^1, \ldots \varphi^s)$ *respectively.*

Then the term

$$\left(\mathfrak{M}_1, m_1, \xi_1, r_1\right); \ldots; \left(\mathfrak{M}_p, m_p, \xi_p, r_p\right); q, s, \varphi, \Phi$$

is called the ***p-ary s-dimensional q-order physical structure*** *on* $\mathfrak{M}_1, \ldots, \mathfrak{M}_p$ *of the dimension* $(m_1, \ldots, m_p)$, *the rank* $(r_1, \ldots, r_p)$, *the characteristics* $\xi_1, \ldots \xi_p$, *the representator* φ *and the verificator* Φ.

Remark 5.4. The general definition of the physical structure is given in [24] (see p. 158). It is explained there the specificity of the binary physical structures that permit finite dimensional displacement groups.

We can use the considered means for analyze these notions. Particularly we can definite a generator of the corresponding physical structures. Consider a map $\varphi : \prod_{i=1}^{p} \mathbb{R}^{n_i} \to \mathbb{R}^s$. Denote by $\langle \Phi, \varphi \rangle$ the superposition of Φ and the transformation that takes the matrixes $\xi_i \in \mathbb{R}^{r_i, n_i}$, $i = 1,...,p$ to the ordering set of p-dimensional matrixes $\varphi^1(\xi_1,...,\xi_p),...,\varphi^s(\xi_1,...,\xi_p)$.

Definition 5.3. *Maps* $\varphi : \prod_{i=1}^{p} \mathbb{R}^{n_i} \to \mathbb{R}^s$ *and* $\Phi : \mathbb{R}^{r_1,...,r_p} \to \mathbb{R}^q$ *are called the **representator** and the **verificator** of p-ary s-dimensional physical structure of the dimension* $(m_1,...,m_p)$*, the rank* $(r_1,...,r_p)$*, and the order q, if*

$$\langle \Phi, \varphi \rangle = 0. \tag{5.4}$$

We can prove the following property with using of the proof method of Theorem 4.2.

Theorem 5.1. *The term* $\mathfrak{M}_1, m_1, \xi_1, r_1 \ ;...;\ \mathfrak{M}_p, m_p, \xi_p, r_p \ ; q, s, \varphi, \Phi$ *is p-ary s-dimensional physical structure of the dimension* $(m_1,...,m_p)$*, the rank* $(r_1,...,r_p)$*, and the order s for all sets* $\mathfrak{M}_1,...,\mathfrak{M}_p$*, the maps* $\xi_i : \mathfrak{M}_i \to \mathbb{R}^{m_i}$, $i = 1,...,p$, *and the pair representator-verificator* (φ, Φ).

The characterization of physical structures does not depend from physical objects classes by Theorem 5.1. It is defined by exclusively numerical objects. Then we can determine equivalence on set of pair of representatot-verificator by analogy to Definition 4.3 and the physical structure generator as the corresponding class of equivalence. Thus the next analysis can be reduced to the finding and classification of physical structure generators.

Remark 5.5. We could extend the physical structure notion for the case of p physical objects classes with m-ary connection, where $p < m$ (see [24]). Then the quantity of the interacting objects is greater than the quantity objects classes. Several objects from one class at least become a party to interaction. There is a vector of natural numbers $(t_1,...,t_p)$ such as $t_1 + ... + t_p = m$. In this situation t_1 objects from first class, t_2 objects from second class etc. interact among themselves. Particularly we had one object class with binary connection for the metric structure on the Euclid plane, i.e. $p = 1$, $m = 2$.

One-dimensional physical structures one the unique set were considered by Kulakov (see [10]). The analogical multidimensional structures were considered by Mikhaylitchenko [14], [15], [16], [21], [23] and Lev [27], [28]. Particularly the completely classification of two-dimensional physical structures is given by Mikhaylitchenko [15], and for three-dimensional physical structures is given by Lev [27]. Multidimensional binary structures and its extensions for greater quantity of objects classes are considered by Mikhaylitchenko [12], [19], [23] and Kyrov [40], [41]. Some general results and the classification of physical structures are given by Mikhaylitchenko (see [24]). The elementary description of binary physical structures theory structures are given by Mikhaylitchenko and Muradov (see [25]).

Next stage of the physical structures extension is the consideration of the verificator and the representator on the complex numbers set. Note also results of Vladimirov in the geometrophysics theory (see [9], [31] – [35]). The corresponding theory of binary systems of complex relations applies in the microphysics. Some application of the complex numbers in the physical structures theory are given also by Mikhaylitchenko (see [19]) and Litvintsev [38]. Mikhaylitchenko and Muradov use the hypercomplex numbers for analysis of physical structures (see [25], [26]). Mikhaylitchenko analyses also group symmetry of physical structures by means of Lie groups and Lie algebras [16], [17], [24].

The next trend of this theory is the renunciation of physical structures on concrete sets and finding the suitable algebraic objects class for physical structures with concrete properties. The first result in this direction is obtained apparently by Ionin (see [37]). We can add also the serious results of Simonov (see [29], [30]). He determinates physical structures by means of the extended multiplication of right-angled matrixes. He proved that the physical structures are connected with algebra of matrixes on the right almost domains. Borodin discovered the connection between (2,2) order physical structures and groups and piles.

The physical structures theory is large relations with nonstandard problems of algebra, geometry, equations theory, and mathematical physics. It is still far from end. Many undecided problems of this direction are given by Mikhaylitchenko (see [24]). Some undecided problems were specified before. The profound analysis of the physical structures theory problems could come nearer to understanding of the general principles underlying physical laws.

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