The generalization of matrix multiplication

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Examples of physical structures

1. Euclidean plane,
2. Ohm's Law.

The axioms of physical structures

1. The algebraic system of axioms,
3. Group identification of physical structures.

Solutions of physical structures

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   а) The physical structure of rank $(2, 2)$,
   б) The physical structure of rank $(3, 2)$,
   в) The physical structure of rank (n, m).

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What is a physical law? On the one hand, it is a restriction? On the other side, on the basis of available data, the law allows us to make a prediction.
But what to consider under relations and what is (characterizes) a stable type?
In 1960s Kulakov suggested the mathematical interpretation of the law concepts. Then several results, concerning the existence and the possible forms of the relations, were achieved.

To characterize the problem, let's turn to some examples. In particular, we are going to have a look at the geometry field. Can we say that all the points are somehow linked with each other? If yes, then how exactly these links are established? By inspecting the distances between the points, we can make a conclusion that the points are located on the same line, space etc. Let's consider the finite set of $\mathfrak{M} = \{i_1, i_2, \dots, i_n\}$, consisting of n randomly located points belonging to the Euclidean plane. Can we say that even with the locations being absolutely random, for every point of $\mathfrak{M}$ there exists a well-defined law? In order to find it, we consider all possible pairs of points of the set $\mathfrak{M}$. The amount of such pairs is $\frac{n(n-1)}{2}$.

Euclidean plane

Matching the distance $\ell_{ik} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}$ to the each pair of points $i, k \in \mathfrak{M}$, we get a set of data, obtained from the experiment, which gives a complete characterization of the set $\mathfrak{M}$. We can present this data in a form of the following matrix:

$$\begin{array}{c|ccccc} & i_1 & i_2 & i_3 & \dots & i_n \\ \hline i_1 & 0 & \ell_{12} & \ell_{13} & \dots & \ell_{1n} \\ i_2 & \ell_{12} & 0 & \ell_{23} & \dots & \ell_{2n} \\ i_3 & \ell_{13} & \ell_{23} & 0 & \dots & \ell_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ i_n & \ell_{1n} & \ell_{2n} & \ell_{3n} & \dots & 0 \end{array}$$

The volume of the simplex

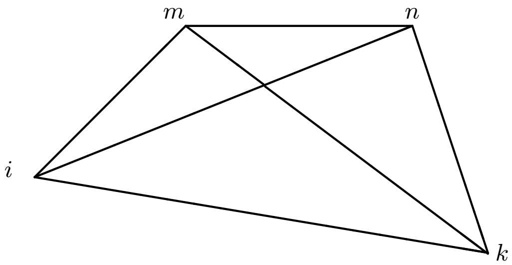
So, for every four points $i, k, m, n \in \mathfrak{M}$ of Euclidean plane $\mathfrak{M}$ there exists a functional dependence between their relative positions, which doesn't depend on the points choice:

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & \ell_{ik}^2 & \ell_{im}^2 & \ell_{in}^2 \\ 1 & \ell_{ik}^2 & 0 & \ell_{km}^2 & \ell_{kn}^2 \\ 1 & \ell_{im}^2 & \ell_{km}^2 & 0 & \ell_{mn}^2 \\ 1 & \ell_{in}^2 & \ell_{kn}^2 & \ell_{mn}^2 & 0 \end{vmatrix} = 0.$$

This determinant is equal (up to a factor) to the square of the three-dimensional simplex, constructed on points $i, k, m, n \in \mathfrak{M}$.

The volume of the simplex

If we have zero three-dimensional volume, then all the points are on the same plane.

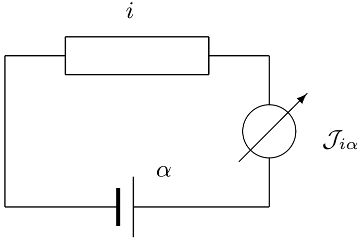


Cayley-Menger Determinant

To expand the previous example, we can take two sets of points $i, j, k, m \in \mathfrak{M}$ and $\alpha, \beta, \gamma, \delta \in \mathfrak{M}$ of Euclidean plane $\mathfrak{M}$ and consider the relative distances between the sets of points with Greek and Latin indexes. For these random sets there exists a functional dependence between their relative distances, which is expressed by the Caly-Manger determinant being zero:

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & \ell_{i\alpha}^2 & \ell_{i\beta}^2 & \ell_{i\gamma}^2 & \ell_{i\delta}^2 \\ 1 & \ell_{j\alpha}^2 & \ell_{j\beta}^2 & \ell_{j\gamma}^2 & \ell_{j\delta}^2 \\ 1 & \ell_{k\alpha}^2 & \ell_{k\beta}^2 & \ell_{k\gamma}^2 & \ell_{k\delta}^2 \\ 1 & \ell_{m\alpha}^2 & \ell_{m\beta}^2 & \ell_{k\gamma}^2 & \ell_{m\delta}^2 \end{vmatrix} = 0.$$

Let's consider the objects of the different kind (contradicting the geometry, where all objects belong to the same set). In this case, two points from two different sets $i \in \mathfrak{M}$, $\alpha \in \mathfrak{N}$ are matched by the result $\mathcal{J}_{i\alpha}$ of the measurement procedure, which serves as an analogue of distance. Consider the set of conductors $\mathfrak{M}$ and the set of current source $\mathfrak{N}$. For optional $i, k, m \in \mathfrak{M}$ and $\alpha, \beta \in \mathfrak{N}$ measure by ammeter the electric current in the following chain:



In this case the ammeter output data $\mathcal{J}_{i\alpha}$ is a distance between the conductor i and the current source α. Consider three independent conductors $i, k, m \in \mathfrak{M}$ and two optional current sources $\alpha, \beta \in \mathfrak{N}$. Measure the six ammeter outputs J. With the sufficient preciseness, we have:

$$\begin{vmatrix} 1 & \mathcal{J}_{i\alpha}^{-1} & \mathcal{J}_{i\beta}^{-1} \\ 1 & \mathcal{J}_{k\alpha}^{-1} & \mathcal{J}_{k\beta}^{-1} \\ 1 & \mathcal{J}_{m\alpha}^{-1} & \mathcal{J}_{m\beta}^{-1} \end{vmatrix} = 0,$$

by these, using the standard points $k, m \in \mathfrak{M}, \beta \in \mathfrak{N}$ we have the well-known Ohm's law for the whole chain

$$\mathcal{J}_{i\alpha} = \frac{\mathcal{E}_\alpha}{R_i + r_\alpha},$$

where $\mathcal{E}_\alpha$ is an electromotive force, r_α is an inner resistance of the current source α and R_i is a resistance of the current i.

In these examples, there are two functions — f and Φ. On the one hand the function

$$f : \mathfrak{M} \times \mathfrak{N} \to \mathbb{R}$$

is an analog of measurement procedure, which assigns to the two elements $i \in \mathfrak{M}$ and $\alpha \in \mathfrak{N}$ of these sets the value $f_{i\alpha}$ of B.
On the other hand, there is a nontrivial function

$$\Phi : \mathbb{R}^{nm} \to 0,$$

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We consider a generalization, and instead of the set of real numbers $\mathbb{R}$, consider an arbitrary set. As a function of Φ, consider the function

$$g : B^{nm-1} \to B,$$

which is the distance across all the restsuch.
Note that the functions f and g are partial, that is, not defined everywhere, but on a subset.

Under **the physical structure of rank** $(m+1, n+1)$ we mean a many-sorted, partial algebraic system $\langle \mathfrak{M}, \mathfrak{N}, B; f, g \rangle$, on the sets $\mathfrak{M}, \mathfrak{N}, B$ acting on them with the operations f, g with axioms.

AI. *For maps* $f : \mathfrak{M} \times \mathfrak{N} \to B,\ \ g : B^{m+mn+n} \to B$, *on arbitrary tuples* $(i_0 i_1 \ldots i_n) \in \mathfrak{M}^{n+1}$ *and* $(\alpha_0 \alpha_1 \ldots \alpha_m) \in \mathfrak{N}^{m+1}$ *the identity*

$$f(i_0, \alpha_0) = g \begin{pmatrix} & f(i_0, \alpha_1) & \cdots & f(i_0, \alpha_m) \\ f(i_1, \alpha_0) & f(i_1, \alpha_1) & \cdots & f(i_1, \alpha_m) \\ \vdots & \vdots & \ddots & \vdots \\ f(i_n, \alpha_0) & f(i_n, \alpha_1) & \cdots & f(i_n, \alpha_m) \end{pmatrix},$$

is correct.

The function g is considered above the elements $f(i_m, \alpha_n) \in B$, constructed above all pairs (i_m, α_n), excluding (i_0, α_0).

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AII. *For all tuples* $(i_1 i_2 \ldots i_n) \in \mathfrak{M}^n$ *and* $(b_1 b_2 \ldots b_n) \in B^n$ *there exists only one element* $\alpha \in \mathfrak{N}$, *for which the following equalities*

$$f(i_k, \alpha) = b_k, \; k \in \{1, 2, \ldots, n\}$$

are true.

AIII. *For all tuples* $(\alpha_1 \alpha_2 \ldots \alpha_m) \in \mathfrak{N}^m$ *and* $(b_1 b_2 \ldots b_m) \in B^m$ *there exists only one element* $i \in \mathfrak{M}$ *for which the following equalities*

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In determining the algebraic system their equivalence is naturally defined.

Definition *For two many–sorted partial algebras* $\langle \mathfrak{M}, \mathfrak{N}, B; f, g\rangle$ *and* $\langle \mathfrak{M}', \mathfrak{N}', B'; f', g'\rangle$ *mapping triples* $\lambda : \mathfrak{M} \to \mathfrak{M}'$, $\chi : \mathfrak{N} \to \mathfrak{N}'$, $\psi : B \to B'$ *define their homomorphism, when the diagrams*

$$\begin{array}{ccc} \mathfrak{M} \times \mathfrak{N} & \xrightarrow{f} & B \\ (\lambda \times \chi) \downarrow & & \downarrow \psi \\ \mathfrak{M}' \times \mathfrak{N}' & \xrightarrow{f'} & B' \end{array} , \qquad \begin{array}{ccc} B^3 & \xrightarrow{g} & B \\ \psi^3 \downarrow & & \downarrow \psi \\ (B')^3 & \xrightarrow{g'} & B' \end{array}$$

are commutative.

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The concept of the physical structure can be defined in an equivalent group form, if together with a map $f : \mathfrak{M} \times \mathfrak{N} \to B$ we also consider two groups of transformations

$$\chi_{a_1,\ldots,a_{mn}} : \mathfrak{M} \to \mathfrak{M},\ \theta_{a_1,\ldots,a_{mn}} : \mathfrak{N} \to \mathfrak{N},$$

depending on the parameters $mn - a_1, \ldots, a_{mn} \in B$.
We assume that this dependence on the parameters is significant, ie it can't be eliminated by any replacement of parameters.
The map f defines the physical structure of rank $(m+1, n+1)$, if it is in harmony with the groups of transformations $\chi_{a_1,\ldots,a_{mn}}$ and $\theta_{a_1,\ldots,a_{mn}}$, for which

$$f(x, y) = f(\chi_{a_1,\ldots,a_{mn}}(x), \theta_{a_1,\ldots,a_{mn}}(y))$$

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The physical structure of rank $(2,2)$

The solution of the physical structure of rank $(2,2)$ over the set $\mathbb{R}$ can be written in the additive $f(i,\alpha) = x_i + \xi_\alpha$ or in the multiplicative form of $f(i,\alpha) = x_i \cdot \xi_\alpha$, for which the corresponding functions Φ can be written as

$$\Phi\left(f(i,\alpha), f(i,\beta), f(j,\alpha), f(j,\beta)\right) = \begin{vmatrix} 0 & 1 & 1 \\ 1 & f(i,\alpha) & f(i,\beta) \\ 1 & f(j,\alpha) & f(j,\beta) \end{vmatrix} = 0,$$

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(Kulakov Y.I., 1968)

The physical structure of rank $(2, 2)$

The solution of the physical structure of rank $(2, 2)$ exists for any arbitrary group $\langle B; \cdot, ^{-1}, e \rangle$. The function f can be written as $f(i, \alpha) = x_i \cdot \xi_\alpha$, and the function g in the form of

$$f(i, \alpha) = g\left(f(i, \beta), f(j, \alpha), f(j, \beta)\right) = f(i, \beta) \cdot f(j, \beta)^{-1} \cdot f(j, \alpha).$$

A group $\langle B; \cdot, ^{-1}, e \rangle$ is called an algebraic system on the set B with one binary operation $(\cdot) : B \times B \to B$, with one unary operation $(^{-1}) : B \to B$, called inverse and one nullary operation e, for which the following axioms are true:

1. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in B$,
2. Nullary operation selects the neutral element $e \in B$ of the group e and $x \cdot e = e \cdot x = x$ for every $x \in B$.
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The physical structure of rank $(2,2)$

The class of physical structures of rank $(2,2)$ is equivalent to the class of all groups $\langle B; \cdot, ^{-1}, e\rangle$. (Ionin V.K., 1990)

Over the set $B = \mathbb{R}$ we can construct only one locally nonisomorphic group — $\mathbb{R}$, so that $f(x,y) = x \cdot y$.

There are two locally not isomorphic groups over the set $B = \mathbb{R}^2$ and they are built with the help of a direct product:
$G_1 = \mathbb{R}_0 \times \mathbb{R}_0$, so that $f_1(x_1,x_2,y_1,y_2) = (x_1 \cdot y_1, x_2 \cdot y_2)$.
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If the physical structure of rank $(2,2)$ can be constructed over an algebra with one binary operation by the group then for building the physical structure of rank $(3,2)$ it requires a richer algebra. Mikchailichenko G.G. showed (1968), that the physical structure of rank $(3,2)$ over $\mathbb{R}$ with the function

$$f(i,\alpha) = x_i\xi_\alpha + \eta_\alpha,$$

can be constructed. Similar solutions can be constructed over an arbitrary field, and even over weaker algebras, such as skew field, Nearfield, Neardomain and a Right neardomain.

Field

F is the field $\langle F;\cdot,+,^{-1},-,1,0\rangle$, with the axioms:
A1. $\langle F;+,-,0\rangle$ is an Abelian group with neutral element $0\in F$,
A2. $\langle F^*;\cdot,^{-1},1\rangle$ is an Abelian group with neutral element $1\in F^*$, where $F^* = F\setminus\{0\}$,
A3. Is a right-sided distributivity $(x+y)z = xz+yz$,
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From the Fields to the Right neardomain

Let's define a skew field through the field. In the transition from the field to the skew field in multiplication the commutativity is lost, ie multiplicative operation can no longer be a commutative group.

Skew field

A2. $\langle F^*; \cdot, ^{-1} \rangle$ — is (an Abelian) group with neutral element $1 \in F$.

In the transition from a field to a Nearfield there is a loss of one of the distributivities, for example, left distributivity.

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From the Fields to the Right neardomain

In the transition from a Nearfield to a Neardomain the requirement for the additive operation to be an abelian group is lost. Now it can be only a loop, ie the requirement of associativity may not be held

$$(x+y)+z \neq x+(y+z),$$

but the following axioms of Neardomain are held:

Neardomain

A1. $\langle F;+,0\rangle$ — (is an Abelian group) is an loop with neutral element $0 \in F$;
A1.2. $a+b=0 \Rightarrow b+a=0$;
A1.3. $(\forall a,b \in F)(\exists\, r_{a,b} \in F^{*}) \quad (x+a)+b = x\cdot r_{a,b} + (a+b)$ for all $x \in F$;
A3.2. $(\forall x \in F) \quad x \cdot 0 = 0$.

Right neardomain

In passing from a Neardomain to a Right neardomain a few more losses happen.
First, if in the additive loop $\langle F;+,0\rangle$ together with the right subtraction

$$(x-y)+y=x,\quad (x+y)-y=x$$

there was also the left subtraction

$$x+(x\leftarrow y)=y,\quad x\leftarrow(x+y)=y,$$

then in the right neardomain $\langle B;\cdot,+,1,A\rangle$ such left subtraction is lost.
Second, if before there was only one zero element 0 for which

$$0\cdot x=0,$$

was fulfilled and now there can be a lot of such zero elements as $A\subset B$:

$$a\cdot x\in A,\quad a\in A.$$

Third - the rest right distributivity is lost.

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If we define the operation of the left inverse in the additive operation

$$L : B \to B \text{ in the form of } L(x) = 0 - x$$

for which $L(x) + x = 0$ is valid and
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$$L : B \to B \text{ in the form of } L(x) = 0 - x$$

for which $L(x) + x = 0$ is valid and
the inverse in the multiplicative operation $E(x) = x^{-1}$ для $x \in B^* = B \setminus A$,
for $\langle B^*; \cdot, ^{-1}, 1\rangle$ is a group, whereas in the right neardomain is:

1. $(x + y) \cdot z = x \cdot EL(y) \cdot L(y \cdot z) + y \cdot z$;
2. $x - y = x \cdot EL^2(y) \cdot y + L(y)$;
3.1. $(x + y) + z = x \cdot (L(z) - y)^{-1} \cdot L(y + z) + (y + z)$, при $y \neq L(z)$
3.2. $(x + L(z)) + z = x \cdot EL^2(z) \cdot z$
satisfied.

The physical structure of rank $(3,2)$

Above the Right neardomain $\langle B; \cdot, +, 1, A\rangle$, with the function of $f(x,y,z) = x \cdot (y - z) + z$ for $x, y, z \in B$ we can construct a group with multiplication:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} f(x_1, y_1, y_2) \\ f(x_2, y_1, y_2) \end{pmatrix}.$$

This is the multiplication group of two-dimensional matrix of columns. In a sense, this is a generalization of matrix multiplication.

Then the physical structure of rank (3,2) on the sets $\mathfrak{M} = B, \mathfrak{N} \subset B^2$ can be constructed using the function $f : \mathfrak{M} \times \mathfrak{N} \to B$ in the form of $f(i,\alpha) = f(x_i, \xi_\alpha, \mu_\alpha)$, where $x_i \in \mathfrak{M}$, $(\xi_\alpha, \mu_\alpha) \in \mathfrak{N}$.

The function of g can be written using the function of f and the generalized matrix multiplication of matrices–columns in the form of:

$$f(i,\alpha) = f\left(f(i,\beta), \begin{pmatrix} f(j,\beta) \\ f(k,\beta) \end{pmatrix}^{-1} \begin{pmatrix} f(j|\alpha) \\ f(k,\alpha) \end{pmatrix}\right).$$

Here are some examples of the right neardomains over $B = \mathbb{R}^2$:

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$$(x_1,x_2)\cdot(y_1,y_2)=(x_1y_1+\varepsilon x_2y_2, x_1y_2+x_2y_1),\quad (\varepsilon=-1,0,1),$$
$$(x_1,x_2)\oplus(y_1,y_2)=(x_1+y_1,x_2+y_2).$$

Solution 2

$$(x_1,x_2)\cdot(y_1,y_2)=(x_1y_1, x_1y_2+x_2y_1^c),\quad c\in[0;1),$$
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Solution 3

$$(x_1,x_2)\cdot(y_1,y_2)=(x_1y_1, x_1y_2+x_2y_1^2+(x_1-1)x_1y_1^2\ln|y_1|),$$
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The physical structure of rank $(3, 2)$

Solution 1

$$(x_1, x_2) \cdot (y_1, y_2) = (x_1 y_1 + \varepsilon x_2 y_2, x_1 y_2 + x_2 y_1), \quad (\varepsilon = -1, 0, 1),$$
$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, x_2 + y_2).$$

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$$(x_1, x_2) \cdot (y_1, y_2) = (x_1 y_1, x_1 y_2 + x_2 y_1^c), \quad c \in [0; 1),$$
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A right neardomain can be written in the equivalent form, but without the additive operation. We construct a unary operation $\varphi_2 : B \to B$ in the form $\varphi_2(x) = x(0-1)+1 = xb+1$, for which the following identities

$$\varphi_2^2(x) = x.$$

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are true.

Theorem. The right neardomain $\langle B; \cdot, +, 1, A\rangle$ and the algebraic system $\langle B; \cdot, ^{-1}, \varphi_2, A\rangle$ are rationally equivalent.
Then the function f of the physical structure of rank $(3,2)$ can be written as:

$$f_{(3,2)}(x, y_1, y_2) = x(y_1 - y_2) + y_2 = \varphi_2(x\varphi_2(y_1 y_2^{-1}))y_2.$$

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The physical structure of rank $(4,2)$

If the algebraic system $\langle B; \cdot, ^{-1}, \varphi_2, A\rangle$ has such unary operation as $\varphi_3 : B \to B$, for which the

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is performed. And if the identity $\varphi_3\varphi_2\varphi_3 = \varphi_2\varphi_3\varphi_2$, correct, then we can construct the function

$$f_{(4,2)}(x, y_1, y_2, y_3) = \varphi_3\left(f_{(3,2)}(x, \varphi_3(y_1 y_3^{-1}), \varphi_3(y_2 y_3^{-1}))\right) y_3.$$

With this function we can construct the group multiplication of the vector–column:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} f_{(4,2)}(x_1, y_1, y_2, y_3) \\ f_{(4,2)}(x_2, y_1, y_2, y_3) \\ f_{(4,2)}(x_3, y_1, y_2, y_3) \end{pmatrix}.$$

Over an arbitrary field F, we can construct a function $\varphi_3(x) = \frac{x-1}{x}$, then the corresponding group will be isomorphic to the group of projective transformations of the given field.

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The physical structure of rank $(n, 2)$

The same situation is repeated in the construction the physical structure of rank $(n+1, 2)$. If the algebraic system $\langle B; \cdot, ^{-1}, \varphi_2, \ldots, \varphi_{n-1} \rangle$ has a unary operation φ_n for which

$$\varphi_n(\varphi_n(x)\varphi_n(y)) = \varphi_n(x\varphi_n(y^{-1}))y.$$

is fulfilled and identities

$$\varphi_n\varphi_i\varphi_n = \varphi_i\varphi_n\varphi_i,$$

for $i \in \{2, \ldots, n-1\}$, are correct, then we can construct the function

$$f_{(n+1,2)}(x, y_1, \ldots, y_n) = \varphi_n \left(f_{(n,2)}(x, \varphi_n(y_1 y_n^{-1}), \ldots, \varphi_n(y_{n-1} y_n^{-1})) \right) y_n.$$

The physical structure of rank $(n+1, n+1)$

Over the set $\mathbb{R}$ there are two and only two non-equivalent solutions for the physical structure of rank $(n+1, n+1)$ (Mikchailichenko G.G., 1970):

$$\begin{cases} f_1(i,\alpha) = x_1\xi_1 + \ldots + x_{n-1}\xi_{n-1} + x_n\xi_n, \\ \Phi_1 = \begin{vmatrix} f(i_1,\alpha_1) & \cdots & f(i_1,\alpha_{n+1}) \\ \vdots & \ddots & \vdots \\ f(i_{n+1},\alpha_1) & \cdots & f(i_{n+1},\alpha_{n+1}) \end{vmatrix} = 0, \end{cases}$$

$$\begin{cases} f_2(i,\alpha) = x_1\xi_1 + \ldots + x_{n-1}\xi_{n-1} + x_n + \xi_n, \\ \Phi_2 = \begin{vmatrix} 0 & 1 & \cdots & 1 \\ 1 & f(i_1,\alpha_1) & \cdots & f(i_1,\alpha_{n+1}) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & f(i_{n+1},\alpha_1) & \cdots & f(i_{n+1},\alpha_{n+1}) \end{vmatrix} = 0, \end{cases}$$

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The physical structure of rank $(n+1, n+1)$

It is well known that using a bilinear function $f_1 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ we can construct the usual matrix multiplication. What can the second solution $f_2 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be connected with?

With the function f_2' in the equivalent record

$$f_2'(x_1, \ldots, x_n, y_1, \ldots, y_n) = \sum_{\mu=1}^{n-1} (x_\mu - x_n)(y_\mu - y_n) + x_n + y_n.$$

it is also possible to construct, but a generalized matrix multiplication $AB = C$ for square matrices A, B, C of dimension n with the usual multiplication rule string into a column for the multiplied matrices:

$$c_{ij} = \left(\begin{array}{ccc} \cdots & \cdots & \cdots \\ \hline a_{i1} & \cdots & a_{in} \\ \hline \cdots & \cdots & \cdots \end{array} \right) \left(\begin{array}{c|c|c} \cdots & b_{1j} & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & b_{nj} & \cdots \end{array} \right)$$

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The generalization of matrix multiplication

You can verify that the product built by using the function f_2' is associative, and the matrices with condition

$$\begin{vmatrix} 0 & 1 & \cdots & 1 \\ 1 & x_{11} & \cdots & x_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nn} \end{vmatrix} \neq 0$$

form a group.

This result can be generalized to an arbitrary case, ie

Theorem (Simonov A.A., 2004) *The physical structure of rank* $(m+1, n+1)$ *with a function*

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Conclusion

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The requirement that a law or relations between the objects of the measured values of the two sets results to:
1. The generalization of matrix multiplication,
2. the emergence of algebraic systems which differ from the fields, near-fields and rings, to the emergence of right neardomain.

Hypothesis
The absence of the right neardomain associativity and partly distributivity is responsible for the violation of CP symmetry in the microcosm, the difference between the right and the left, and a violation of the superposition of quantum states at higher energies.

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Thank you very much!
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